Mathematica 11.3 Integration Test Results

Test results for the 413 problems in "1.2.2.3 (d+e x^2)^m (a+b x^2+c x^4)^p.m"

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+b\,x^2}{\sqrt{1-b^2\,x^4}}\,dx$$

Optimal (type 4, 16 leaves, 2 steps):

$$\frac{\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{b}\;x\right],-1\right]}{\sqrt{b}}$$

Result (type 4, 27 leaves):

$$-\frac{i\;\text{EllipticE}\left[i\;\text{ArcSinh}\left[\sqrt{-b}\;x\right],-1\right]}{\sqrt{-b}}$$

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-b\,x^2}{\sqrt{1-b^2\,x^4}}\,dx$$

Optimal (type 4, 35 leaves, 5 steps):

$$-\frac{\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{b}\;x\right],-1\right]}{\sqrt{b}}+\frac{2\;\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{b}\;x\right],-1\right]}{\sqrt{b}}$$

Result (type 4, 46 leaves):

$$\frac{i\left(\text{EllipticE}\left[i\;\text{ArcSinh}\left[\sqrt{-b}\;x\right],-1\right]-2\;\text{EllipticF}\left[i\;\text{ArcSinh}\left[\sqrt{-b}\;x\right],-1\right]\right)}{\sqrt{-b}}$$

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+b\,x^2}{\sqrt{-1+b^2\,x^4}}\,dx$$

Optimal (type 4, 43 leaves, 3 steps):

$$\frac{\sqrt{1-b^2\,x^4}\;\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{b}\;x\right],-1\right]}{\sqrt{b}\;\sqrt{-1+b^2\,x^4}}$$

Result (type 4, 54 leaves):

$$-\frac{i \sqrt{1-b^2 x^4}\ \text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{-b}\ x\right],-1\right]}{\sqrt{-b}\ \sqrt{-1+b^2 x^4}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-b x^2}{\sqrt{-1+b^2 x^4}}\, dx$$

Optimal (type 4, 89 leaves, 6 steps):

$$-\frac{\sqrt{1-b^2 x^4}\ \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{b}\ x\right],-1\right]}{\sqrt{b}\ \sqrt{-1+b^2 x^4}}+\frac{2 \sqrt{1-b^2 x^4}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{b}\ x\right],-1\right]}{\sqrt{b}\ \sqrt{-1+b^2 x^4}}$$

Result (type 4, 73 leaves):

$$\left(i \sqrt{1-b^2 x^4}\ \left(\text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{-b}\ x\right],-1\right]-2\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{-b}\ x\right],-1\right]\right)\right)\Big/ \left(\sqrt{-b}\ \sqrt{-1+b^2 x^4}\right)$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1-b x^2}{\sqrt{1+b^2 x^4}}\, dx$$

Optimal (type 4, 89 leaves, 1 step):

$$-\frac{x \sqrt{1+b^2 x^4}}{1+b x^2}+\frac{\left(1+b x^2\right) \sqrt{\frac{1+b^2 x^4}{\left(1+b x^2\right)^2}}\ \text{EllipticE}\left[2\ \text{ArcTan}\left[\sqrt{b}\ x\right],\frac{1}{2}\right]}{\sqrt{b}\ \sqrt{1+b^2 x^4}}$$

Result (type 4, 52 leaves):

$$-\frac{1}{\sqrt{i b}}\left(\text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{i b}\ x\right],-1\right]-(1-i)\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{i b}\ x\right],-1\right]\right)$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+b x^2}{\sqrt{1+b^2 x^4}}\, dx$$

Optimal (type 4, 152 leaves, 3 steps):

$$\frac{x\sqrt{1+b^2x^4}}{1+bx^2}-\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{1+b^2x^4}}+\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{1+b^2x^4}}$$

Result (type 4, 51 leaves):

$$\frac{1}{\sqrt{i\,b}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]-(1+i)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]\right)$$

Problem 20: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1-b\,x^2}{\sqrt{-1-b^2x^4}}\,dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}+\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{-1-b^2x^4}}$$

Result (type 4, 79 leaves):

$$-\left(\left(\sqrt{1+b^2x^4}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]-(1-i)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{i\,b}\,x\right],-1\right]\right)\right)\Big/\left(\sqrt{i\,b}\,\sqrt{-1-b^2x^4}\right)\right)$$

Problem 21: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1+b\,x^2}{\sqrt{-1-b^2x^4}}\,dx$$

Optimal (type 4, 156 leaves, 3 steps):

$$-\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}-\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{-1-b^2x^4}}+\frac{\left(1+bx^2\right)\sqrt{\frac{1+b^2x^4}{\left(1+bx^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\sqrt{b}\,x\right],\frac{1}{2}\right]}{\sqrt{b}\,\sqrt{-1-b^2x^4}}$$

Result (type 4, 78 leaves):

$$\left(\sqrt{1+b^2 x^4}\ \left(\text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{i\ b}\ x\right],-1\right]-(1+i)\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{i\ b}\ x\right],-1\right]\right)\right)\Big/\left(\sqrt{i\ b}\ \sqrt{-1-b^2 x^4}\right)$$

Problem 23: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+c^2 x^2}{\sqrt{1-c^4 x^4}}\, dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\frac{\text{EllipticE}[\text{ArcSin}[c\ x],-1]}{c}$$

Result (type 4, 31 leaves):

$$-\frac{i\ \text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{-c^2}\ x\right],-1\right]}{\sqrt{-c^2}}$$

Problem 25: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1-c^2 x^2}{\sqrt{1-c^4 x^4}}\, dx$$

Optimal (type 4, 23 leaves, 5 steps):

$$-\frac{\text{EllipticE}[\text{ArcSin}[c\ x],-1]}{c}+\frac{2\ \text{EllipticF}[\text{ArcSin}[c\ x],-1]}{c}$$

Result (type 4, 52 leaves):

$$\frac{1}{\sqrt{-c^2}}i\left(\text{EllipticE}\left[i\ \text{ArcSinh}\left[\sqrt{-c^2}\ x\right],-1\right]-2\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[\sqrt{-c^2}\ x\right],-1\right]\right)$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e x^2}{d^2+b x^2+e^2 x^4}\, dx$$

Optimal (type 3, 82 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{-b+2 d e}-2 e x}{\sqrt{b+2 d e}}\right]}{\sqrt{b+2 d e}}+\frac{\text{ArcTan}\left[\frac{\sqrt{-b+2 d e}+2 e x}{\sqrt{b+2 d e}}\right]}{\sqrt{b+2 d e}}$$

Result (type 3, 181 leaves):

$$\frac{1}{\sqrt{2}\,\sqrt{b^2-4 d^2 e^2}}\left(\frac{\left(-b+2 d e+\sqrt{b^2-4 d^2 e^2}\right) \text{ArcTan}\left[\frac{\sqrt{2}\, e x}{\sqrt{b-\sqrt{b^2-4 d^2 e^2}}}\right]}{\sqrt{b-\sqrt{b^2-4 d^2 e^2}}}+\frac{\left(b-2 d e+\sqrt{b^2-4 d^2 e^2}\right) \text{ArcTan}\left[\frac{\sqrt{2}\, e x}{\sqrt{b+\sqrt{b^2-4 d^2 e^2}}}\right]}{\sqrt{b+\sqrt{b^2-4 d^2 e^2}}}\right)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e x^2}{d^2+f x^2+e^2 x^4} \, dx$$

Optimal (type 3, 82 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2 d e-f}-2 e x}{\sqrt{2 d e+f}}\right]}{\sqrt{2 d e+f}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2 d e-f}+2 e x}{\sqrt{2 d e+f}}\right]}{\sqrt{2 d e+f}}$$

Result (type 3, 181 leaves):

$$\left(\frac{\left(2 d e-f+\sqrt{-4 d^2 e^2+f^2}\right) \text{ArcTan}\left[\frac{\sqrt{2}\, e x}{\sqrt{f-\sqrt{-4 d^2 e^2+f^2}}}\right]}{\sqrt{f-\sqrt{-4 d^2 e^2+f^2}}}+\frac{\left(-2 d e+f+\sqrt{-4 d^2 e^2+f^2}\right) \text{ArcTan}\left[\frac{\sqrt{2}\, e x}{\sqrt{f+\sqrt{-4 d^2 e^2+f^2}}}\right]}{\sqrt{f+\sqrt{-4 d^2 e^2+f^2}}}\right) \Big/ \left(\sqrt{2}\,\sqrt{-4 d^2 e^2+f^2}\right)$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e x^2}{d^2-b x^2+e^2 x^4} \, dx$$

Optimal (type 3, 78 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{b+2 d e}-2 e x}{\sqrt{b-2 d e}}\right]}{\sqrt{b-2 d e}}-\frac{\text{ArcTanh}\left[\frac{\sqrt{b+2 d e}+2 e x}{\sqrt{b-2 d e}}\right]}{\sqrt{b-2 d e}}$$

Result (type 3, 189 leaves):

$$\frac{1}{\sqrt{2}\,\sqrt{b^2-4\,d^2\,e^2}}\left(\frac{\left(b+2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-b-\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{-b-\sqrt{b^2-4\,d^2\,e^2}}}+\frac{\left(-b-2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-b+\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{-b+\sqrt{b^2-4\,d^2\,e^2}}}\right)$$

Problem 29: Result more than twice size of optimal antiderivative.

$$\int \frac{d+e\,x^2}{d^2-f\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 86 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2\,d\,e+f}-2\,e\,x}{\sqrt{2\,d\,e-f}}\right]}{\sqrt{2\,d\,e-f}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2\,d\,e+f}+2\,e\,x}{\sqrt{2\,d\,e-f}}\right]}{\sqrt{2\,d\,e-f}}$$

Result (type 3, 189 leaves):

$$\left(\frac{\left(2\,d\,e+f+\sqrt{-4\,d^2\,e^2+f^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-f-\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{-f-\sqrt{-4\,d^2\,e^2+f^2}}}+\frac{\left(-2\,d\,e-f+\sqrt{-4\,d^2\,e^2+f^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-f+\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{-f+\sqrt{-4\,d^2\,e^2+f^2}}}\right)\Big/\left(\sqrt{2}\,\sqrt{-4\,d^2\,e^2+f^2}\right)$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \frac{d-e\,x^2}{d^2+b\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 78 leaves, 3 steps):

$$-\frac{\text{Log}\left[d-\sqrt{-b+2\,d\,e}\,x+e\,x^2\right]}{2\,\sqrt{-b+2\,d\,e}}+\frac{\text{Log}\left[d+\sqrt{-b+2\,d\,e}\,x+e\,x^2\right]}{2\,\sqrt{-b+2\,d\,e}}$$

Result (type 3, 182 leaves):

$$\frac{1}{\sqrt{2}\,\sqrt{b^2-4\,d^2\,e^2}}\left(\frac{\left(b+2\,d\,e-\sqrt{b^2-4\,d^2\,e^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{b-\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{b-\sqrt{b^2-4\,d^2\,e^2}}}-\frac{\left(b+2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{b+\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{b+\sqrt{b^2-4\,d^2\,e^2}}}\right)$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{d-e\,x^2}{d^2+f\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 78 leaves, 3 steps):

$$-\frac{\text{Log}\left[d-\sqrt{2\,d\,e-f}\,x+e\,x^2\right]}{2\,\sqrt{2\,d\,e-f}}+\frac{\text{Log}\left[d+\sqrt{2\,d\,e-f}\,x+e\,x^2\right]}{2\,\sqrt{2\,d\,e-f}}$$

Result (type 3, 182 leaves):

$$\left(\frac{\left(2\,d\,e+f-\sqrt{-4\,d^2\,e^2+f^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{f-\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{f-\sqrt{-4\,d^2\,e^2+f^2}}}-\frac{\left(2\,d\,e+f+\sqrt{-4\,d^2\,e^2+f^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{f+\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{f+\sqrt{-4\,d^2\,e^2+f^2}}}\right)\Big/\left(\sqrt{2}\,\sqrt{-4\,d^2\,e^2+f^2}\right)$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int\frac{d-e\,x^2}{d^2-b\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 70 leaves, 3 steps):

$$-\frac{\text{Log}\left[d-\sqrt{b+2\,d\,e}\,x+e\,x^2\right]}{2\,\sqrt{b+2\,d\,e}}+\frac{\text{Log}\left[d+\sqrt{b+2\,d\,e}\,x+e\,x^2\right]}{2\,\sqrt{b+2\,d\,e}}$$

Result (type 3, 190 leaves):

$$\frac{1}{\sqrt{2}\,\sqrt{b^2-4\,d^2\,e^2}}\left(-\frac{\left(b-2\,d\,e+\sqrt{b^2-4\,d^2\,e^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-b-\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{-b-\sqrt{b^2-4\,d^2\,e^2}}}+\frac{\left(b-2\,d\,e-\sqrt{b^2-4\,d^2\,e^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-b+\sqrt{b^2-4\,d^2\,e^2}}}\right]}{\sqrt{-b+\sqrt{b^2-4\,d^2\,e^2}}}\right)$$

Problem 33: Result more than twice size of optimal antiderivative.

$$\int\frac{d-e\,x^2}{d^2-f\,x^2+e^2\,x^4}\,dx$$

Optimal (type 3, 70 leaves, 3 steps):

$$-\frac{\text{Log}\left[d-\sqrt{2\,d\,e+f}\,x+e\,x^2\right]}{2\,\sqrt{2\,d\,e+f}}+\frac{\text{Log}\left[d+\sqrt{2\,d\,e+f}\,x+e\,x^2\right]}{2\,\sqrt{2\,d\,e+f}}$$

Result (type 3, 190 leaves):

$$\left(-\frac{\left(-2\,d\,e+f+\sqrt{-4\,d^2\,e^2+f^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-f-\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{-f-\sqrt{-4\,d^2\,e^2+f^2}}}+\frac{\left(-2\,d\,e+f-\sqrt{-4\,d^2\,e^2+f^2}\right)\,\text{ArcTan}\left[\frac{\sqrt{2}\,e\,x}{\sqrt{-f+\sqrt{-4\,d^2\,e^2+f^2}}}\right]}{\sqrt{-f+\sqrt{-4\,d^2\,e^2+f^2}}}\right)\Big/\left(\sqrt{2}\,\sqrt{-4\,d^2\,e^2+f^2}\right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int\frac{a+b\,x^2}{a^2+\left(-1+2\,a\,b\right)\,x^2+b^2\,x^4}\,dx$$

Optimal (type 3, 60 leaves, 5 steps):

$$\frac{\text{ArcTanh}\left[\frac{1-2\,b\,x}{\sqrt{1-4\,a\,b}}\right]}{\sqrt{1-4\,a\,b}}-\frac{\text{ArcTanh}\left[\frac{1+2\,b\,x}{\sqrt{1-4\,a\,b}}\right]}{\sqrt{1-4\,a\,b}}$$

Result (type 3, 138 leaves):

$$\frac{\frac{\left(1+\sqrt{1-4 a b}\right) \text{ArcTan}\left[\frac{b x}{\sqrt{-\frac{1}{2}+a b-\frac{1}{2} \sqrt{1-4 a b}}}\right]}{\sqrt{-1+2 a b-\sqrt{1-4 a b}}}+\frac{\left(-1+\sqrt{1-4 a b}\right) \text{ArcTan}\left[\frac{\sqrt{2} b x}{\sqrt{-1+2 a b+\sqrt{1-4 a b}}}\right]}{\sqrt{-1+2 a b+\sqrt{1-4 a b}}}}{\sqrt{2-8 a b}}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{1+2 x^2}{1+b x^2+4 x^4} \, dx$$

Optimal (type 3, 62 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{4-b}-4 x}{\sqrt{4+b}}\right]}{\sqrt{4+b}}+\frac{\text{ArcTan}\left[\frac{\sqrt{4-b}+4 x}{\sqrt{4+b}}\right]}{\sqrt{4+b}}$$

Result (type 3, 126 leaves):

$$\frac{\frac{\left(4-b+\sqrt{-16+b^2}\right) \text{ArcTan}\left[\frac{2 \sqrt{2} x}{\sqrt{b-\sqrt{-16+b^2}}}\right]}{\sqrt{b-\sqrt{-16+b^2}}}+\frac{\left(-4+b+\sqrt{-16+b^2}\right) \text{ArcTan}\left[\frac{2 \sqrt{2} x}{\sqrt{b+\sqrt{-16+b^2}}}\right]}{\sqrt{b+\sqrt{-16+b^2}}}}{\sqrt{2} \sqrt{-16+b^2}}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{1+2 x^2}{1-b x^2+4 x^4} \, dx$$

Optimal (type 3, 66 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{4+b}-4 x}{\sqrt{4-b}}\right]}{\sqrt{4-b}}+\frac{\text{ArcTan}\left[\frac{\sqrt{4+b}+4 x}{\sqrt{4-b}}\right]}{\sqrt{4-b}}$$

Result (type 3, 134 leaves):

$$\frac{\frac{\left(4+b+\sqrt{-16+b^2}\right) \text{ArcTan}\left[\frac{2 \sqrt{2} x}{\sqrt{-b-\sqrt{-16+b^2}}}\right]}{\sqrt{-b-\sqrt{-16+b^2}}}+\frac{\left(-4-b+\sqrt{-16+b^2}\right) \text{ArcTan}\left[\frac{2 \sqrt{2} x}{\sqrt{-b+\sqrt{-16+b^2}}}\right]}{\sqrt{-b+\sqrt{-16+b^2}}}}{\sqrt{2} \sqrt{-16+b^2}}$$

Problem 44: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2 x^2}{1+3 x^2+4 x^4} \, dx$$

Optimal (type 3, 38 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-4x}{\sqrt{7}}\right]}{\sqrt{7}}+\frac{\text{ArcTan}\left[\frac{1+4x}{\sqrt{7}}\right]}{\sqrt{7}}$$

Result (type 3, 97 leaves):

$$\frac{\left(-i+\sqrt{7}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(3-i\sqrt{7}\right)}}\right]}{\sqrt{42-14\,i\sqrt{7}}}+\frac{\left(i+\sqrt{7}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(3+i\sqrt{7}\right)}}\right]}{\sqrt{42+14\,i\sqrt{7}}}$$

Problem 45: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+2x^2}{1+2x^2+4x^4}\,dx$$

Optimal (type 3, 48 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2\sqrt{2}\,x}{\sqrt{3}}\right]}{\sqrt{6}}+\frac{\text{ArcTan}\left[\frac{1+2\sqrt{2}\,x}{\sqrt{3}}\right]}{\sqrt{6}}$$

Result (type 3, 99 leaves):

$$\frac{\left(-i+\sqrt{3}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{1-i\sqrt{3}}}\right]}{2\sqrt{3\left(1-i\sqrt{3}\right)}}+\frac{\left(i+\sqrt{3}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{1+i\sqrt{3}}}\right]}{2\sqrt{3\left(1+i\sqrt{3}\right)}}$$

Problem 46: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1+2x^2}{1+x^2+4x^4}\,dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3}-4x}{\sqrt{5}}\right]}{\sqrt{5}}+\frac{\text{ArcTan}\left[\frac{\sqrt{3}+4x}{\sqrt{5}}\right]}{\sqrt{5}}$$

Result (type 3, 97 leaves):

$$\frac{\left(-3\,i+\sqrt{15}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{15}\right)}}\right]}{\sqrt{30-30\,i\sqrt{15}}}+\frac{\left(3\,i+\sqrt{15}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{15}\right)}}\right]}{\sqrt{30+30\,i\sqrt{15}}}$$

Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx$$

Optimal (type 3, 46 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{5}-4x}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{\sqrt{5}+4x}{\sqrt{3}}\right]}{\sqrt{3}}$$

Result (type 3, 101 leaves):

$$\frac{\left(-5\,i+\sqrt{15}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{15}\right)}}\right]}{\sqrt{30\left(-1-i\sqrt{15}\right)}}+\frac{\left(5\,i+\sqrt{15}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{15}\right)}}\right]}{\sqrt{30\left(-1+i\sqrt{15}\right)}}$$

Problem 49: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\sqrt{3}-2\sqrt{2}\,x\right]}{\sqrt{2}}+\frac{\text{ArcTan}\left[\sqrt{3}+2\sqrt{2}\,x\right]}{\sqrt{2}}$$

Result (type 3, 99 leaves):

$$\frac{\left(-3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right]}{2\sqrt{3\left(-1-i\sqrt{3}\right)}}+\frac{\left(3\,i+\sqrt{3}\right)\text{ArcTan}\left[\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right]}{2\sqrt{3\left(-1+i\sqrt{3}\right)}}$$

Problem 65: Result more than twice size of optimal antiderivative.

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx$$

Optimal (type 3, 14 leaves, 3 steps):

$$\frac{\text{ArcTanh}\left[\sqrt{2}\,x\right]}{\sqrt{2}}$$

Result (type 3, 32 leaves):

$$\frac{-\text{Log}\left[\sqrt{2}-2x\right]+\text{Log}\left[\sqrt{2}+2x\right]}{2\sqrt{2}}$$

Problem 73: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx$$

Optimal (type 3, 38 leaves, 5 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2x}{\sqrt{3}}\right]}{\sqrt{3}}+\frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{3}}\right]}{\sqrt{3}}$$

Result (type 3, 99 leaves):

$$\frac{\left(-i+\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right]}{\sqrt{6\left(1-i\sqrt{3}\right)}}+\frac{\left(i+\sqrt{3}\right)\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right]}{\sqrt{6\left(1+i\sqrt{3}\right)}}$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \frac{1-x^2}{1+b\,x^2+x^4}\,dx$$

Optimal (type 3, 62 leaves, 3 steps):

$$-\frac{\text{Log}\left[1-\sqrt{2-b}\;x+x^2\right]}{2\sqrt{2-b}}+\frac{\text{Log}\left[1+\sqrt{2-b}\;x+x^2\right]}{2\sqrt{2-b}}$$

Result (type 3, 125 leaves):

$$\frac{\frac{\left(2+b-\sqrt{-4+b^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\;x}{\sqrt{b-\sqrt{-4+b^2}}}\right]}{\sqrt{b-\sqrt{-4+b^2}}}-\frac{\left(2+b+\sqrt{-4+b^2}\right)\text{ArcTan}\left[\frac{\sqrt{2}\;x}{\sqrt{b+\sqrt{-4+b^2}}}\right]}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\;\sqrt{-4+b^2}}$$

Problem 88: Result more than twice size of optimal antiderivative.

$$\int \frac{1-x^2}{1-2\,x^2+x^4}\,dx$$

Optimal (type 3, 2 leaves, 3 steps):

$$\text{ArcTanh}[x]$$

Result (type 3, 19 leaves):

$$-\frac{1}{2}\text{Log}[1-x]+\frac{1}{2}\text{Log}[1+x]$$

Problem 92: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int -\frac{1+3 x^2}{1+2 x^2+9 x^4} \, dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{1-3 x}{\sqrt{2}}\right]}{2 \sqrt{2}}-\frac{\text{ArcTan}\left[\frac{1+3 x}{\sqrt{2}}\right]}{2 \sqrt{2}}$$

Result (type 3, 99 leaves):

$$-\frac{\left(-i+\sqrt{2}\right) \text{ArcTan}\left[\frac{3 x}{\sqrt{1-2 i \sqrt{2}}}\right]}{2 \sqrt{2 \left(1-2 i \sqrt{2}\right)}}-\frac{\left(i+\sqrt{2}\right) \text{ArcTan}\left[\frac{3 x}{\sqrt{1+2 i \sqrt{2}}}\right]}{2 \sqrt{2 \left(1+2 i \sqrt{2}\right)}}$$

Problem 93: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+3 x^2}{-1-2 x^2-9 x^4} \, dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\text{ArcTan}\left[\frac{1-3 x}{\sqrt{2}}\right]}{2 \sqrt{2}}-\frac{\text{ArcTan}\left[\frac{1+3 x}{\sqrt{2}}\right]}{2 \sqrt{2}}$$

Result (type 3, 99 leaves):

$$-\frac{\left(-i+\sqrt{2}\right) \text{ArcTan}\left[\frac{3 x}{\sqrt{1-2 i \sqrt{2}}}\right]}{2 \sqrt{2 \left(1-2 i \sqrt{2}\right)}}-\frac{\left(i+\sqrt{2}\right) \text{ArcTan}\left[\frac{3 x}{\sqrt{1+2 i \sqrt{2}}}\right]}{2 \sqrt{2 \left(1+2 i \sqrt{2}\right)}}$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{1+x^2+x^4} \, dx$$

Optimal (type 3, 83 leaves, 9 steps):

$$-\frac{(a+b) \text{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{2 \sqrt{3}}+\frac{(a+b) \text{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{2 \sqrt{3}}-\frac{1}{4} (a-b) \text{Log}\left[1-x+x^2\right]+\frac{1}{4} (a-b) \text{Log}\left[1+x+x^2\right]$$

Result (type 3, 97 leaves):

$$\frac{\left(2 i a+\left(-i+\sqrt{3}\right) b\right) \text{ArcTan}\left[\frac{1}{2} \left(-i+\sqrt{3}\right) x\right]}{\sqrt{6+6 i \sqrt{3}}}+\frac{\left(-2 i a+\left(i+\sqrt{3}\right) b\right) \text{ArcTan}\left[\frac{1}{2} \left(i+\sqrt{3}\right) x\right]}{\sqrt{6-6 i \sqrt{3}}}$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{\left(1+x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 119 leaves, 10 steps):

$$\frac{x\left(a+b-(a-2 b) x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{(4 a+b)\,\text{ArcTan}\left[\frac{1-2 x}{\sqrt{3}}\right]}{12 \sqrt{3}}+\frac{(4 a+b)\,\text{ArcTan}\left[\frac{1+2 x}{\sqrt{3}}\right]}{12 \sqrt{3}}-\frac{1}{8}(2 a-b)\,\text{Log}\left[1-x+x^2\right]+\frac{1}{8}(2 a-b)\,\text{Log}\left[1+x+x^2\right]$$

Result (type 3, 147 leaves):

$$\frac{x\left(a+b-a x^2+2 b x^2\right)}{6\left(1+x^2+x^4\right)}-\frac{\left(\left(-11\, i+\sqrt{3}\right) a-2\left(-2\, i+\sqrt{3}\right) b\right)\,\text{ArcTan}\left[\frac{1}{2}\left(-i+\sqrt{3}\right) x\right]}{6 \sqrt{6+6\, i \sqrt{3}}}-\frac{\left(\left(11\, i+\sqrt{3}\right) a-2\left(2\, i+\sqrt{3}\right) b\right)\,\text{ArcTan}\left[\frac{1}{2}\left(i+\sqrt{3}\right) x\right]}{6 \sqrt{6-6\, i \sqrt{3}}}$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{2+x^2+x^4}\,dx$$

Optimal (type 3, 234 leaves, 9 steps):

$$-\frac{1}{2} \sqrt{\frac{1}{14}\left(-1+2 \sqrt{2}\right)}\left(a+\sqrt{2}\, b\right)\,\text{ArcTan}\left[\frac{\sqrt{-1+2 \sqrt{2}}-2 x}{\sqrt{1+2 \sqrt{2}}}\right]+\frac{1}{2} \sqrt{\frac{1}{14}\left(-1+2 \sqrt{2}\right)}\left(a+\sqrt{2}\, b\right)\,\text{ArcTan}\left[\frac{\sqrt{-1+2 \sqrt{2}}+2 x}{\sqrt{1+2 \sqrt{2}}}\right]-\frac{\left(a-\sqrt{2}\, b\right)\,\text{Log}\left[\sqrt{2}-\sqrt{-1+2 \sqrt{2}}\, x+x^2\right]}{4 \sqrt{2\left(-1+2 \sqrt{2}\right)}}+\frac{\left(a-\sqrt{2}\, b\right)\,\text{Log}\left[\sqrt{2}+\sqrt{-1+2 \sqrt{2}}\, x+x^2\right]}{4 \sqrt{2\left(-1+2 \sqrt{2}\right)}}$$

Result (type 3, 111 leaves):

$$\frac{\left(-2\, i\, a+\left(i+\sqrt{7}\right) b\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1-i \sqrt{7}\right)}}\right]}{\sqrt{14-14\, i \sqrt{7}}}+\frac{\left(2\, i\, a+\left(-i+\sqrt{7}\right) b\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1+i \sqrt{7}\right)}}\right]}{\sqrt{14+14\, i \sqrt{7}}}$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{\left(2+x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 316 leaves, 10 steps):

$$\frac{x\left(3 a+2 b-(a-4 b) x^2\right)}{28\left(2+x^2+x^4\right)}-\frac{1}{56} \sqrt{\frac{1}{14}\left(-1+2 \sqrt{2}\right)}\left(\left(11-\sqrt{2}\right) a-\left(2-4 \sqrt{2}\right) b\right) \text{ArcTan}\left[\frac{\sqrt{-1+2 \sqrt{2}}-2 x}{\sqrt{1+2 \sqrt{2}}}\right]+\frac{1}{56} \sqrt{\frac{1}{14}\left(-1+2 \sqrt{2}\right)}\left(\left(11-\sqrt{2}\right) a-\left(2-4 \sqrt{2}\right) b\right) \text{ArcTan}\left[\frac{\sqrt{-1+2 \sqrt{2}}+2 x}{\sqrt{1+2 \sqrt{2}}}\right]-\frac{\left(11 a+\sqrt{2}(a-4 b)-2 b\right) \text{Log}\left[\sqrt{2}-\sqrt{-1+2 \sqrt{2}}\, x+x^2\right]}{112 \sqrt{2\left(-1+2 \sqrt{2}\right)}}+\frac{\left(\left(11+\sqrt{2}\right) a-2\left(b+2 \sqrt{2}\, b\right)\right) \text{Log}\left[\sqrt{2}+\sqrt{-1+2 \sqrt{2}}\, x+x^2\right]}{112 \sqrt{2\left(-1+2 \sqrt{2}\right)}}$$

Result (type 3, 165 leaves):

$$\frac{-a x\left(-3+x^2\right)+2 b\left(x+2 x^3\right)}{28\left(2+x^2+x^4\right)}-\frac{\left(\left(23 i+\sqrt{7}\right) a-4\left(2 i+\sqrt{7}\right) b\right) \text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1-i \sqrt{7}\right)}}\right]}{28 \sqrt{14-14 i \sqrt{7}}}-\frac{\left(\left(-23 i+\sqrt{7}\right) a-4\left(-2 i+\sqrt{7}\right) b\right) \text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}\left(1+i \sqrt{7}\right)}}\right]}{28 \sqrt{14+14 i \sqrt{7}}}$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}\, x^2+x^4} \, dx$$

Optimal (type 3, 160 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2+\sqrt{2}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right]}{2\sqrt{2+\sqrt{2}}}-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\,\text{Log}\left[1-\sqrt{2+\sqrt{2}}\,x+x^2\right]+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\,\text{Log}\left[1+\sqrt{2+\sqrt{2}}\,x+x^2\right]$$

Result (type 3, 53 leaves):

$$\frac{\sqrt{-1-i}\,\text{ArcTan}\left[\frac{2^{1/4}x}{\sqrt{-1-i}}\right]+\sqrt{-1+i}\,\text{ArcTan}\left[\frac{2^{1/4}x}{\sqrt{-1+i}}\right]}{2^{3/4}}$$

Problem 103: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx$$

Optimal (type 3, 172 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2-\sqrt{2}}}+\frac{\text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right]}{2\sqrt{2-\sqrt{2}}}-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\,\text{Log}\left[1-\sqrt{2-\sqrt{2}}\,x+x^2\right]+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\,\text{Log}\left[1+\sqrt{2-\sqrt{2}}\,x+x^2\right]$$

Result (type 3, 53 leaves):

$$\frac{\sqrt{1-i}\,\text{ArcTan}\left[\frac{2^{1/4}x}{\sqrt{1-i}}\right]+\sqrt{1+i}\,\text{ArcTan}\left[\frac{2^{1/4}x}{\sqrt{1+i}}\right]}{2^{3/4}}$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2a-x^2}{a^2-a x^2+x^4}\,dx$$

Optimal (type 3, 114 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\sqrt{3}-\frac{2x}{\sqrt{a}}\right]}{2\sqrt{a}}+\frac{\text{ArcTan}\left[\sqrt{3}+\frac{2x}{\sqrt{a}}\right]}{2\sqrt{a}}-\frac{\sqrt{3}\,\text{Log}\left[a-\sqrt{3}\,\sqrt{a}\,x+x^2\right]}{4\sqrt{a}}+\frac{\sqrt{3}\,\text{Log}\left[a+\sqrt{3}\,\sqrt{a}\,x+x^2\right]}{4\sqrt{a}}$$

Result (type 3, 115 leaves):

$$\frac{1}{2\sqrt{6}\sqrt{a}}(-1)^{1/4}\left(-\sqrt{i+\sqrt{3}}\left(3 i+\sqrt{3}\right)\text{ArcTan}\left[\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt{a}}\right]+\sqrt{-i+\sqrt{3}}\left(-3 i+\sqrt{3}\right)\text{ArcTanh}\left[\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt{a}}\right]\right)$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}x^2+x^4}\,dx$$

Optimal (type 3, 122 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\sqrt{3}-\frac{2x}{a^{1/4}}\right]}{2a^{1/4}}+\frac{\text{ArcTan}\left[\sqrt{3}+\frac{2x}{a^{1/4}}\right]}{2a^{1/4}}-\frac{\sqrt{3}\,\text{Log}\left[\sqrt{a}-\sqrt{3}a^{1/4}x+x^2\right]}{4a^{1/4}}+\frac{\sqrt{3}\,\text{Log}\left[\sqrt{a}+\sqrt{3}a^{1/4}x+x^2\right]}{4a^{1/4}}$$

Result (type 3, 115 leaves):

$$\frac{1}{2\sqrt{6}a^{1/4}}(-1)^{1/4}\left(-\sqrt{i+\sqrt{3}}\left(3 i+\sqrt{3}\right)\text{ArcTan}\left[\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}a^{1/4}}\right]+\sqrt{-i+\sqrt{3}}\left(-3 i+\sqrt{3}\right)\text{ArcTanh}\left[\frac{(1+i)x}{\sqrt{i+\sqrt{3}}a^{1/4}}\right]\right)$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx$$

Optimal (type 3, 124 leaves, 9 steps):

$$-\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{b^{1/3}-2x}{\sqrt{3}b^{1/3}}\right]}{2b^{1/3}}+\frac{\sqrt{3}\,\text{ArcTan}\left[\frac{b^{1/3}+2x}{\sqrt{3}b^{1/3}}\right]}{2b^{1/3}}-\frac{\text{Log}\left[b^{2/3}-b^{1/3}x+x^2\right]}{4b^{1/3}}+\frac{\text{Log}\left[b^{2/3}+b^{1/3}x+x^2\right]}{4b^{1/3}}$$

Result (type 3, 115 leaves):

$$\frac{1}{2\sqrt{6}b^{1/3}}(-1)^{1/4}\left(\sqrt{-i+\sqrt{3}}\left(-3 i+\sqrt{3}\right)\text{ArcTan}\left[\frac{(1+i)x}{\sqrt{i+\sqrt{3}}b^{1/3}}\right]-\sqrt{i+\sqrt{3}}\left(3 i+\sqrt{3}\right)\text{ArcTanh}\left[\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}b^{1/3}}\right]\right)$$

Problem 109: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{a^2 - a x^2 + x^4} \, dx$$

Optimal (type 3, 136 leaves, 9 steps):

$$-\frac{(A + a B) \text{ArcTan}\left[\sqrt{3} - \frac{2 x}{\sqrt{a}}\right]}{2 a^{3/2}} + \frac{(A + a B) \text{ArcTan}\left[\sqrt{3} + \frac{2 x}{\sqrt{a}}\right]}{2 a^{3/2}} - \frac{(A - a B) \text{Log}\left[a - \sqrt{3} \sqrt{a} x + x^2\right]}{4 \sqrt{3} a^{3/2}} + \frac{(A - a B) \text{Log}\left[a + \sqrt{3} \sqrt{a} x + x^2\right]}{4 \sqrt{3} a^{3/2}}$$

Result (type 3, 130 leaves):

$$\frac{1}{\sqrt{6} a^{3/2}} (-1)^{1/4} \left(\frac{\left(-2 i A + \left(-i + \sqrt{3}\right) a B\right) \text{ArcTan}\left[\frac{(1+i) x}{\sqrt{-i+\sqrt{3}} \sqrt{a}}\right]}{\sqrt{-i + \sqrt{3}}} - \frac{\left(2 i A + \left(i + \sqrt{3}\right) a B\right) \text{ArcTanh}\left[\frac{(1+i) x}{\sqrt{i+\sqrt{3}} \sqrt{a}}\right]}{\sqrt{i + \sqrt{3}}}\right)$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A + B x^2}{a - \sqrt{a} x^2 + x^4} \, dx$$

Optimal (type 3, 160 leaves, 9 steps):

$$-\frac{\left(A + \sqrt{a} B\right) \text{ArcTan}\left[\sqrt{3} - \frac{2 x}{a^{1/4}}\right]}{2 a^{3/4}} + \frac{\left(A + \sqrt{a} B\right) \text{ArcTan}\left[\sqrt{3} + \frac{2 x}{a^{1/4}}\right]}{2 a^{3/4}} - \frac{\left(A - \sqrt{a} B\right) \text{Log}\left[\sqrt{a} - \sqrt{3} a^{1/4} x + x^2\right]}{4 \sqrt{3} a^{3/4}} + \frac{\left(A - \sqrt{a} B\right) \text{Log}\left[\sqrt{a} + \sqrt{3} a^{1/4} x + x^2\right]}{4 \sqrt{3} a^{3/4}}$$

Result (type 3, 138 leaves):

$$\frac{1}{\sqrt{6} a^{3/4}} (-1)^{1/4} \left(\frac{\left(-2 i A + \left(-i + \sqrt{3}\right) \sqrt{a} B\right) \text{ArcTan}\left[\frac{(1+i) x}{\sqrt{-i+\sqrt{3}} a^{1/4}}\right]}{\sqrt{-i + \sqrt{3}}} - \frac{\left(2 i A + \left(i + \sqrt{3}\right) \sqrt{a} B\right) \text{ArcTanh}\left[\frac{(1+i) x}{\sqrt{i+\sqrt{3}} a^{1/4}}\right]}{\sqrt{i + \sqrt{3}}}\right)$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{a-\sqrt{a c}\, x^2+c x^4}\, dx$$

Optimal (type 3, 414 leaves, 9 steps):

$$-\frac{\left(\sqrt{a}\, B+A \sqrt{c}\right) \text{ArcTan}\left[\frac{\sqrt{2 \sqrt{a}\, \sqrt{c}+\sqrt{a c}}-2 \sqrt{c}\, x}{\sqrt{2 \sqrt{a}\, \sqrt{c}-\sqrt{a c}}}\right]}{2 \sqrt{a}\, \sqrt{c}\, \sqrt{2 \sqrt{a}\, \sqrt{c}-\sqrt{a c}}}+\frac{\left(\sqrt{a}\, B+A \sqrt{c}\right) \text{ArcTan}\left[\frac{\sqrt{2 \sqrt{a}\, \sqrt{c}+\sqrt{a c}}+2 \sqrt{c}\, x}{\sqrt{2 \sqrt{a}\, \sqrt{c}-\sqrt{a c}}}\right]}{2 \sqrt{a}\, \sqrt{c}\, \sqrt{2 \sqrt{a}\, \sqrt{c}-\sqrt{a c}}}-\frac{\left(A-\frac{\sqrt{a}\, B}{\sqrt{c}}\right) \text{Log}\left[\sqrt{a}-\sqrt{2 \sqrt{a}\, \sqrt{c}+\sqrt{a c}}\, x+\sqrt{c}\, x^2\right]}{4 \sqrt{a}\, \sqrt{2 \sqrt{a}\, \sqrt{c}+\sqrt{a c}}}+\frac{\left(A-\frac{\sqrt{a}\, B}{\sqrt{c}}\right) \text{Log}\left[\sqrt{a}+\sqrt{2 \sqrt{a}\, \sqrt{c}+\sqrt{a c}}\, x+\sqrt{c}\, x^2\right]}{4 \sqrt{a}\, \sqrt{2 \sqrt{a}\, \sqrt{c}+\sqrt{a c}}}$$

Result (type 3, 247 leaves):

$$\frac{1}{\sqrt{6}\, \sqrt{a}\, c}\left(\frac{\left(\sqrt{3}\, \sqrt{a}\, B \sqrt{c}-i \left(2 A c+B \sqrt{a c}\right)\right) \text{ArcTan}\left[\frac{\sqrt{2}\, \sqrt{c}\, x}{\sqrt{-i \sqrt{3}\, \sqrt{a}\, \sqrt{c}-\sqrt{a c}}}\right]}{\sqrt{-i \sqrt{3}\, \sqrt{a}\, \sqrt{c}-\sqrt{a c}}}+\frac{\left(\sqrt{3}\, \sqrt{a}\, B \sqrt{c}+i \left(2 A c+B \sqrt{a c}\right)\right) \text{ArcTan}\left[\frac{\sqrt{2}\, \sqrt{c}\, x}{\sqrt{i \sqrt{3}\, \sqrt{a}\, \sqrt{c}-\sqrt{a c}}}\right]}{\sqrt{i \sqrt{3}\, \sqrt{a}\, \sqrt{c}-\sqrt{a c}}}\right)$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{A+B x^2}{a-\sqrt{a}\, \sqrt{c}\, x^2+c x^4}\, dx$$

Optimal (type 3, 234 leaves, 9 steps):

$$-\frac{\left(\sqrt{a}\, B+A \sqrt{c}\right) \text{ArcTan}\left[\sqrt{3}-\frac{2 c^{1/4} x}{a^{1/4}}\right]}{2 a^{3/4} c^{3/4}}+\frac{\left(\sqrt{a}\, B+A \sqrt{c}\right) \text{ArcTan}\left[\sqrt{3}+\frac{2 c^{1/4} x}{a^{1/4}}\right]}{2 a^{3/4} c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}\, B}{\sqrt{c}}\right) \text{Log}\left[\sqrt{a}-\sqrt{3}\, a^{1/4} c^{1/4} x+\sqrt{c}\, x^2\right]}{4 \sqrt{3}\, a^{3/4} c^{1/4}}+\frac{\left(A-\frac{\sqrt{a}\, B}{\sqrt{c}}\right) \text{Log}\left[\sqrt{a}+\sqrt{3}\, a^{1/4} c^{1/4} x+\sqrt{c}\, x^2\right]}{4 \sqrt{3}\, a^{3/4} c^{1/4}}$$

Result (type 3, 163 leaves):

$$\frac{1}{\sqrt{6}\, a^{3/4}\, c^{3/4}}(-1)^{1/4}\left(\frac{\left(\left(-\mathrm{i}+\sqrt{3}\right)\sqrt{a}\; B-2\,\mathrm{i}\,A\sqrt{c}\right)\mathrm{ArcTan}\left[\frac{(1+\mathrm{i})\, c^{1/4}\, x}{\sqrt{-\mathrm{i}+\sqrt{3}}\; a^{1/4}}\right]}{\sqrt{-\mathrm{i}+\sqrt{3}}}-\frac{\left(\left(\mathrm{i}+\sqrt{3}\right)\sqrt{a}\; B+2\,\mathrm{i}\,A\sqrt{c}\right)\mathrm{ArcTanh}\left[\frac{(1+\mathrm{i})\, c^{1/4}\, x}{\sqrt{\mathrm{i}+\sqrt{3}}\; a^{1/4}}\right]}{\sqrt{\mathrm{i}+\sqrt{3}}}\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\, dx$$

Optimal (type 4, 96 leaves, 4 steps):

$$-\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}\;\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\sqrt{\frac{2}{1+\sqrt{13}}}\; x\right],\frac{1}{6}\left(-7-\sqrt{13}\right)\right]+\sqrt{7+2\sqrt{13}}\;\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\sqrt{\frac{2}{1+\sqrt{13}}}\; x\right],\frac{1}{6}\left(-7-\sqrt{13}\right)\right]$$

Result (type 4, 103 leaves):

$$-\frac{1}{\sqrt{2\left(1+\sqrt{13}\right)}}\mathrm{i}\left(\left(1+\sqrt{13}\right)\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{2}{-1+\sqrt{13}}}\; x\right],\frac{1}{6}\left(-7+\sqrt{13}\right)\right]-\left(-5+\sqrt{13}\right)\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{\frac{2}{-1+\sqrt{13}}}\; x\right],\frac{1}{6}\left(-7+\sqrt{13}\right)\right]\right)$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\, dx$$

Optimal (type 4, 25 leaves, 5 steps):

$$-\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{x}{\sqrt{3}}\right],-3\right]+4\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{x}{\sqrt{3}}\right],-3\right]$$

Result (type 4, 19 leaves):

$$-\mathrm{i}\sqrt{3}\;\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}[x],-\frac{1}{3}\right]$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3 - x^2}{\sqrt{3 + 3 x^2 - x^4}} \, dx$$

Optimal (type 4, 96 leaves, 4 steps):

$$-\sqrt{\frac{1}{2} \left(-3 + \sqrt{21}\right)} \text{ EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{2}{3 + \sqrt{21}}} x\right], \frac{1}{2} \left(-5 - \sqrt{21}\right)\right] + \sqrt{9 + 2 \sqrt{21}} \text{ EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{3 + \sqrt{21}}} x\right], \frac{1}{2} \left(-5 - \sqrt{21}\right)\right]$$

Result (type 4, 103 leaves):

$$-\frac{1}{\sqrt{2 \left(3 + \sqrt{21}\right)}} i \left(\left(3 + \sqrt{21}\right) \text{ EllipticE}\left[i \text{ ArcSinh}\left[\sqrt{\frac{2}{-3 + \sqrt{21}}} x\right], \frac{1}{2} \left(-5 + \sqrt{21}\right)\right] - \left(-3 + \sqrt{21}\right) \text{ EllipticF}\left[i \text{ ArcSinh}\left[\sqrt{\frac{2}{-3 + \sqrt{21}}} x\right], \frac{1}{2} \left(-5 + \sqrt{21}\right)\right]\right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3 - x^2}{\sqrt{3 - x^2 - x^4}} \, dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$-\sqrt{\frac{1}{2} \left(1 + \sqrt{13}\right)} \text{ EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-1 + \sqrt{13}}} x\right], \frac{1}{6} \left(-7 + \sqrt{13}\right)\right] + \sqrt{5 + 2 \sqrt{13}} \text{ EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-1 + \sqrt{13}}} x\right], \frac{1}{6} \left(-7 + \sqrt{13}\right)\right]$$

Result (type 4, 107 leaves):

$$-\frac{1}{\sqrt{2 \left(-1 + \sqrt{13}\right)}} i \left(\left(-1 + \sqrt{13}\right) \text{ EllipticE}\left[i \text{ ArcSinh}\left[\sqrt{\frac{2}{1 + \sqrt{13}}} x\right], -\frac{7}{6} - \frac{\sqrt{13}}{6}\right] - \left(-7 + \sqrt{13}\right) \text{ EllipticF}\left[i \text{ ArcSinh}\left[\sqrt{\frac{2}{1 + \sqrt{13}}} x\right], -\frac{7}{6} - \frac{\sqrt{13}}{6}\right]\right)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx$$

Optimal (type 4, 27 leaves, 4 steps):

$$-\sqrt{3}\ \text{EllipticE}\left[\text{ArcSin}[x],-\frac{1}{3}\right]+2\sqrt{3}\ \text{EllipticF}\left[\text{ArcSin}[x],-\frac{1}{3}\right]$$

Result (type 4, 35 leaves):

$$-i\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],-3\right]+2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{3}}\right],-3\right]\right)$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$-\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}\ \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-3+\sqrt{21}}}\ x\right],\frac{1}{2}\left(-5+\sqrt{21}\right)\right]+$$
$$\sqrt{3+2\sqrt{21}}\ \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{-3+\sqrt{21}}}\ x\right],\frac{1}{2}\left(-5+\sqrt{21}\right)\right]$$

Result (type 4, 107 leaves):

$$-\frac{1}{\sqrt{2\left(-3+\sqrt{21}\right)}}\,i\left(\left(-3+\sqrt{21}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{3+\sqrt{21}}}\ x\right],-\frac{5}{2}-\frac{\sqrt{21}}{2}\right]-\right.$$
$$\left.\left(-9+\sqrt{21}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{2}{3+\sqrt{21}}}\ x\right],-\frac{5}{2}-\frac{\sqrt{21}}{2}\right]\right)$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 296 leaves, 3 steps):

$$\frac{2\sqrt{c}\,x\sqrt{a+b x^2+c x^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{1}{\sqrt{a+b x^2+c x^4}}2\,a^{1/4}c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]+$$

$$\left(\left(b+2\sqrt{a}\sqrt{c}-\sqrt{b^2-4 a c}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(2\,a^{1/4}c^{1/4}\sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 187 leaves):

$$-\frac{1}{\sqrt{a+b x^2+c x^4}}2\,\mathrm{i}\,\sqrt{2}\,a\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}$$

$$\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\,x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]$$

Problem 150: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e x^2\right)^4}{\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 388 leaves, 6 steps):

$$\frac{e^2\left(42 c d^2-5 a e^2\right)x\sqrt{a+c x^4}}{21 c^2}+\frac{4 d e^3 x^3\sqrt{a+c x^4}}{5 c}+$$

$$\frac{e^4 x^5\sqrt{a+c x^4}}{7 c}+\frac{4 d e\left(5 c d^2-3 a e^2\right)x\sqrt{a+c x^4}}{5 c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{1}{5 c^{7/4}\sqrt{a+c x^4}}$$

$$4\,a^{1/4}d e\left(5 c d^2-3 a e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(\left(105 c^2 d^4+420\sqrt{a}\,c^{3/2}d^3 e-210 a c d^2 e^2-252 a^{3/2}\sqrt{c}\,d e^3+25 a^2 e^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\right.$$

$$\left.\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(210\,a^{1/4}c^{9/4}\sqrt{a+c x^4}\right)$$

Result (type 4, 298 leaves):

$$\frac{1}{105\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,c^2\sqrt{a+c\,x^4}}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,e^2\,x\left(-25\,a^2\,e^2+2\,a\,c\left(105\,d^2+42\,d\,e\,x^2-5\,e^2\,x^4\right)+3\,c^2\,x^4\left(70\,d^2+28\,d\,e\,x^2+5\,e^2\,x^4\right)\right)-84\,\sqrt{a}\,\sqrt{c}\,d\,e\left(-5\,c\,d^2+3\,a\,e^2\right)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]+\left(-105\,i\,c^2\,d^4-420\,\sqrt{a}\,c^{3/2}\,d^3\,e+210\,i\,a\,c\,d^2\,e^2+252\,a^{3/2}\,\sqrt{c}\,d\,e^3-25\,i\,a^2\,e^4\right)\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]\right)$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e\,x^2\right)^3}{\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 326 leaves, 5 steps):

$$\frac{d\,e^2\,x\,\sqrt{a+c\,x^4}}{c}+\frac{e^3\,x^3\,\sqrt{a+c\,x^4}}{5\,c}+\frac{3\,e\left(5\,c\,d^2-a\,e^2\right)x\,\sqrt{a+c\,x^4}}{5\,c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{1}{5\,c^{7/4}\sqrt{a+c\,x^4}}3\,a^{1/4}\,e\left(5\,c\,d^2-a\,e^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+\frac{1}{10\,c^{7/4}\sqrt{a+c\,x^4}}a^{1/4}\left(15\,c\,d^2\,e-3\,a\,e^3+\frac{5\sqrt{c}\,d\left(c\,d^2-a\,e^2\right)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 235 leaves):

$$\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ \sqrt{c}\ e^2\,x\,\left(5\,d+e\,x^2\right)\,\left(a+c\,x^4\right)-\right.$$

$$3\,\sqrt{a}\ e\,\left(-5\,c\,d^2+a\,e^2\right)\,\sqrt{1+\frac{c\,x^4}{a}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]+$$

$$\left(-5\,i\,c^{3/2}\,d^3-15\,\sqrt{a}\ c\,d^2\,e+5\,i\,a\,\sqrt{c}\ d\,e^2+3\,a^{3/2}\,e^3\right)\,\sqrt{1+\frac{c\,x^4}{a}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]\right)\Big/\left(5\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ c^{3/2}\,\sqrt{a+c\,x^4}\right)$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e\,x^2\right)^2}{\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 264 leaves, 4 steps):

$$\frac{e^2\,x\,\sqrt{a+c\,x^4}}{3\,c}+\frac{2\,d\,e\,x\,\sqrt{a+c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\ x^2\right)}-\frac{1}{c^{3/4}\,\sqrt{a+c\,x^4}}$$

$$2\,a^{1/4}\,d\,e\,\left(\sqrt{a}+\sqrt{c}\ x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]+$$

$$\left(\left(3\,c\,d^2+6\,\sqrt{a}\ \sqrt{c}\ d\,e-a\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\ x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(6\,a^{1/4}\,c^{5/4}\,\sqrt{a+c\,x^4}\right)$$

Result (type 4, 195 leaves):

$$\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ e^2 x\left(a+c x^4\right)+6\sqrt{a}\ \sqrt{c}\ d\, e\sqrt{1+\frac{c x^4}{a}}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]+\right.$$

$$\left.i\left(-3 c d^2+6 i\sqrt{a}\ \sqrt{c}\ d\, e+a e^2\right)\sqrt{1+\frac{c x^4}{a}}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]\right)\Big/\left(3\right.$$

$$\left.\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ c\sqrt{a+c x^4}\right)$$

Problem 153: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e x^2}{\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 226 leaves, 3 steps):

$$\frac{e x\sqrt{a+c x^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\ x^2\right)}-\frac{a^{1/4} e\left(\sqrt{a}+\sqrt{c}\ x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{c^{3/4}\sqrt{a+c x^4}}+\frac{1}{2 c^{3/4}\sqrt{a+c x^4}}$$

$$a^{1/4}\left(\frac{\sqrt{c}\ d}{\sqrt{a}}+e\right)\left(\sqrt{a}+\sqrt{c}\ x^2\right)\sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]$$

Result (type 4, 131 leaves):

$$\left(\sqrt{1+\frac{c x^4}{a}}\left(\sqrt{a}\ e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]+\right.\right.$$

$$\left.\left.\left(-i\sqrt{c}\ d-\sqrt{a}\ e\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]\right)\right)\Big/\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ \sqrt{c}\ \sqrt{a+c x^4}\right)$$

Problem 154: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e x^2\right)\sqrt{a+c x^4}}\,dx$$

Optimal (type 4, 334 leaves, 3 steps):

$$\frac{\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{d}\,\sqrt{c\,d^2+a\,e^2}}+\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+c\,x^4}}-$$

$$\left(a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,c^{1/4}\,d\left(c\,d^2-a\,e^2\right)\sqrt{a+c\,x^4}\right)$$

Result (type 4, 95 leaves):

$$-\frac{i\,\sqrt{1+\frac{c\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{i\,\sqrt{a}\,e}{\sqrt{c}\,d},i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,x\right],-1\right]}{\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\,d\,\sqrt{a+c\,x^4}}$$

Problem 155: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e\,x^2\right)^2\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 581 leaves, 6 steps):

$$-\frac{\sqrt{c}\ e\,x\,\sqrt{a+c\,x^4}}{2\,d\,\left(c\,d^2+a\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\ x^2\right)}+$$

$$\frac{e^2\,x\,\sqrt{a+c\,x^4}}{2\,d\,\left(c\,d^2+a\,e^2\right)\,\left(d+e\,x^2\right)}+\frac{\sqrt{e}\ \left(3\,c\,d^2+a\,e^2\right)\,\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\ x}{\sqrt{d}\ \sqrt{e}\ \sqrt{a+c\,x^4}}\right]}{4\,d^{3/2}\,\left(c\,d^2+a\,e^2\right)^{3/2}}+$$

$$\left(a^{1/4}\,c^{1/4}\,e\,\left(\sqrt{a}+\sqrt{c}\ x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(2\,d\,\left(c\,d^2+a\,e^2\right)\,\sqrt{a+c\,x^4}\right)+\frac{c^{1/4}\,\left(\sqrt{a}+\sqrt{c}\ x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]}{2\,a^{1/4}\,d\,\left(\sqrt{c}\ d-\sqrt{a}\ e\right)\,\sqrt{a+c\,x^4}}-$$

$$\left(\left(\sqrt{c}\ d+\sqrt{a}\ e\right)\,\left(3\,c\,d^2+a\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\ x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\ x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\ d-\sqrt{a}\ e\right)^2}{4\,\sqrt{a}\ \sqrt{c}\ d\,e},\,2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,a^{1/4}\,c^{1/4}\,d^2\,\left(\sqrt{c}\ d-\sqrt{a}\ e\right)\,\left(c\,d^2+a\,e^2\right)\,\sqrt{a+c\,x^4}\right)$$

Result (type 4, 522 leaves) :

$$\frac{1}{2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, d^2\left(c d^2+a e^2\right)\left(d+e x^2\right)\sqrt{a+c x^4}}\left(a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, d e^2 x+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, c d e^2 x^5-\right.$$

$$\sqrt{a}\,\sqrt{c}\, d e\left(d+e x^2\right)\sqrt{1+\frac{c x^4}{a}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+$$

$$\sqrt{c}\, d\left(i\sqrt{c}\, d+\sqrt{a}\, e\right)\left(d+e x^2\right)\sqrt{1+\frac{c x^4}{a}}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-$$

$$3 i c d^3\sqrt{1+\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-$$

$$i a d e^2\sqrt{1+\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-$$

$$3 i c d^2 e x^2\sqrt{1+\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-$$

$$\left. i a e^3 x^2\sqrt{1+\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{i\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

Problem 156: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)^3\sqrt{a+c x^4}}\, dx$$

Optimal (type 4, 729 leaves, 7 steps):

$$-\frac{3\sqrt{c}\,e\,\left(3\,c\,d^2+a\,e^2\right)\,x\,\sqrt{a+c\,x^4}}{8\,d^2\,\left(c\,d^2+a\,e^2\right)^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2\,x\,\sqrt{a+c\,x^4}}{4\,d\,\left(c\,d^2+a\,e^2\right)\,\left(d+e\,x^2\right)^2}+$$

$$\frac{3\,e^2\,\left(3\,c\,d^2+a\,e^2\right)\,x\,\sqrt{a+c\,x^4}}{8\,d^2\,\left(c\,d^2+a\,e^2\right)^2\,\left(d+e\,x^2\right)}+\frac{3\,\sqrt{e}\,\left(5\,c^2\,d^4+2\,a\,c\,d^2\,e^2+a^2\,e^4\right)\,\text{ArcTan}\left[\frac{\sqrt{c\,d^2+a\,e^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+c\,x^4}}\right]}{16\,d^{5/2}\,\left(c\,d^2+a\,e^2\right)^{5/2}}+$$

$$\left(3\,a^{1/4}\,c^{1/4}\,e\,\left(3\,c\,d^2+a\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(8\,d^2\,\left(c\,d^2+a\,e^2\right)^2\,\sqrt{a+c\,x^4}\right)+$$

$$\left(c^{1/4}\,\left(4\,c\,d^2-\sqrt{a}\,\sqrt{c}\,d\,e+3\,a\,e^2\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/\left(8\,a^{1/4}\,d^2\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2+a\,e^2\right)\,\sqrt{a+c\,x^4}\right)-$$

$$\left(3\,\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\,\left(5\,c^2\,d^4+2\,a\,c\,d^2\,e^2+a^2\,e^4\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d\,e},\,2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{2}\right]\right)\Big/$$

$$\left(32\,a^{1/4}\,c^{1/4}\,d^3\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\,\left(c\,d^2+a\,e^2\right)^2\,\sqrt{a+c\,x^4}\right)$$

Result (type 4, 332 leaves) :

$$\left(\frac{d e^2 x \left(a+c x^4\right) \left(a e^2 \left(5 d+3 e x^2\right)+c d^2 \left(11 d+9 e x^2\right)\right)}{\left(d+e x^2\right)^2}+\frac{1}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}\right.$$

$$\sqrt{1+\frac{c x^4}{a}} \left(-3 \sqrt{a} \sqrt{c} d e \left(3 c d^2+a e^2\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]+\right.$$

$$i \left(\sqrt{c} d \left(7 c^{3/2} d^3-9 i \sqrt{a} c d^2 e+a \sqrt{c} d e^2-3 i a^{3/2} e^3\right)\right.$$

$$\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]-3 \left(5 c^2 d^4+2 a c d^2 e^2+a^2 e^4\right)$$

$$\left.\left.\left.\text{EllipticPi}\left[-\frac{i \sqrt{a} e}{\sqrt{c} d}, i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right],-1\right]\right)\right)\right) \Big/ \left(8 d^3 \left(c d^2+a e^2\right)^2 \sqrt{a+c x^4}\right)$$

Problem 157: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^3}{\sqrt{a-c x^4}} \, dx$$

Optimal (type 4, 213 leaves, 8 steps):

$$-\frac{d e^2 x \sqrt{a-c x^4}}{c}-\frac{e^3 x^3 \sqrt{a-c x^4}}{5 c}+$$

$$\frac{3 a^{3/4} e \left(5 c d^2+a e^2\right) \sqrt{1-\frac{c x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{5 c^{7/4} \sqrt{a-c x^4}}+\frac{1}{5 c^{7/4} \sqrt{a-c x^4}}$$

$$a^{3/4} \left(\frac{5 \sqrt{c} d \left(c d^2+a e^2\right)}{\sqrt{a}}-3 e \left(5 c d^2+a e^2\right)\right) \sqrt{1-\frac{c x^4}{a}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]$$

Result (type 4, 232 leaves):

$$\left(\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2} e^2 x \left(5 d+e x^2\right)\left(a-c x^4\right)-3 i \sqrt{a}\, e\left(5 c d^2+a e^2\right)\sqrt{1-\frac{c x^4}{a}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+i\left(-5 c^{3/2} d^3+15 \sqrt{a}\, c d^2 e-5 a \sqrt{c}\, d e^2+3 a^{3/2} e^3\right)\sqrt{1-\frac{c x^4}{a}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)\Big/\left(5\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, c^{3/2}\sqrt{a-c x^4}\right)$$

Problem 158: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^2}{\sqrt{a-c x^4}}\, dx$$

Optimal (type 4, 162 leaves, 7 steps):

$$-\frac{e^2 x \sqrt{a-c x^4}}{3 c}+\frac{2 a^{3/4} d e \sqrt{1-\frac{c x^4}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{c^{3/4}\sqrt{a-c x^4}}+\frac{1}{3 c^{5/4}\sqrt{a-c x^4}} a^{1/4}\left(3 c d^2-6 \sqrt{a}\sqrt{c}\, d e+a e^2\right)\sqrt{1-\frac{c x^4}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]$$

Result (type 4, 192 leaves):

$$\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, e^2 x\left(-a+c x^4\right)-6 i \sqrt{a}\sqrt{c}\, d e\sqrt{1-\frac{c x^4}{a}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-i\left(3 c d^2-6 \sqrt{a}\sqrt{c}\, d e+a e^2\right)\sqrt{1-\frac{c x^4}{a}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)\Big/\left(3\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, c\sqrt{a-c x^4}\right)$$

Problem 159: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x^2}{\sqrt{a-c x^4}}\, dx$$

Optimal (type 4, 124 leaves, 6 steps):

$$\frac{a^{3/4}\, e\, \sqrt{1-\frac{c x^4}{a}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{c^{3/4}\sqrt{a-c x^4}}+\frac{a^{3/4}\left(\frac{\sqrt{c}\, d}{\sqrt{a}}-e\right)\sqrt{1-\frac{c x^4}{a}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{c^{3/4}\sqrt{a-c x^4}}$$

Result (type 4, 127 leaves):

$$\left(i\sqrt{1-\frac{c x^4}{a}}\left(\sqrt{a}\, e\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+\left(\sqrt{c}\, d-\sqrt{a}\, e\right)\text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)\right)\Big/\left(\sqrt{a}\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{a-c x^4}\right)$$

Problem 160: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)\sqrt{a-c x^4}}\, dx$$

Optimal (type 4, 72 leaves, 2 steps):

$$\frac{a^{1/4}\sqrt{1-\frac{c x^4}{a}}\; \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d},\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{c^{1/4}\, d\sqrt{a-c x^4}}$$

Result (type 4, 91 leaves):

$$-\frac{i\sqrt{1-\frac{c x^4}{a}}\; \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, d\sqrt{a-c x^4}}$$

Problem 161: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)^2\sqrt{a-c x^4}}\, dx$$

Optimal (type 4, 299 leaves, 10 steps):

$$-\frac{e^2 x \sqrt{a-c x^4}}{2 d \left(c d^2-a e^2\right) \left(d+e x^2\right)}-\frac{a^{3/4} c^{1/4} e \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{2 d \left(c d^2-a e^2\right) \sqrt{a-c x^4}}-$$

$$\frac{a^{1/4} c^{1/4} \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{2 d \left(\sqrt{c}\, d+\sqrt{a}\, e\right) \sqrt{a-c x^4}}+$$

$$\frac{a^{1/4} \left(3 c d^2-a e^2\right) \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d},\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{2 c^{1/4} d^2 \left(c d^2-a e^2\right) \sqrt{a-c x^4}}$$

Result (type 4, 508 leaves):

$$\frac{1}{2 \sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, d^2 \left(c d^2-a e^2\right) \left(d+e x^2\right) \sqrt{a-c x^4}}\left(-a \sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, d e^2 x+\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, c d e^2 x^5+\right.$$

$$i \sqrt{a} \sqrt{c}\, d e \left(d+e x^2\right) \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-$$

$$i \sqrt{c}\, d \left(-\sqrt{c}\, d+\sqrt{a}\, e\right) \left(d+e x^2\right) \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-$$

$$3 i c d^3 \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+$$

$$i a d e^2 \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]-$$

$$3 i c d^2 e x^2 \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+$$

$$\left.i a e^3 x^2 \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)$$

Problem 162: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)^3 \sqrt{a-c x^4}}\, dx$$

Optimal (type 4, 425 leaves, 11 steps):

$$-\frac{e^2 x \sqrt{a-c x^4}}{4 d \left(c d^2-a e^2\right) \left(d+e x^2\right)^2}-\frac{3 e^2 \left(3 c d^2-a e^2\right) x \sqrt{a-c x^4}}{8 d^2 \left(c d^2-a e^2\right)^2 \left(d+e x^2\right)}-$$

$$\frac{3 a^{3/4} c^{1/4} e \left(3 c d^2-a e^2\right) \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]}{8 d^2 \left(c d^2-a e^2\right)^2 \sqrt{a-c x^4}}-$$

$$\left(a^{1/4} c^{1/4} \left(7 c d^2-2 \sqrt{a} \sqrt{c} d e-3 a e^2\right) \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]\right) \Big/$$

$$\left(8 d^2 \left(\sqrt{c} d+\sqrt{a} e\right) \left(c d^2-a e^2\right) \sqrt{a-c x^4}\right)+$$

$$\left(3 a^{1/4} \left(5 c^2 d^4-2 a c d^2 e^2+a^2 e^4\right) \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a} e}{\sqrt{c} d},\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]\right) \Big/$$

$$\left(8 c^{1/4} d^3 \left(c d^2-a e^2\right)^2 \sqrt{a-c x^4}\right)$$

Result (type 4, 321 leaves):

$$\left(\frac{d e^2 x \left(a-c x^4\right) \left(a e^2 \left(5 d+3 e x^2\right)-c d^2 \left(11 d+9 e x^2\right)\right)}{\left(d+e x^2\right)^2}-\frac{1}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right.$$

$$i \sqrt{1-\frac{c x^4}{a}} \left(3 \sqrt{a} \sqrt{c} d e \left(-3 c d^2+a e^2\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]+\right.$$

$$\left(-7 c^2 d^4+9 \sqrt{a} c^{3/2} d^3 e+a c d^2 e^2-3 a^{3/2} \sqrt{c} d e^3\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]+$$

$$\left.\left.3 \left(5 c^2 d^4-2 a c d^2 e^2+a^2 e^4\right) \text{EllipticPi}\left[-\frac{\sqrt{a} e}{\sqrt{c} d}, i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]\right)\right) \Big/$$

$$\left(8 d^3 \left(c d^2-a e^2\right)^2 \sqrt{a-c x^4}\right)$$

Problem 163: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)^4 \sqrt{a-c x^4}}\, dx$$

Optimal (type 4, 563 leaves, 12 steps):

$$-\frac{e^2 x \sqrt{a-c x^4}}{6 d \left(c d^2-a e^2\right) \left(d+e x^2\right)^3}-\frac{5 e^2 \left(3 c d^2-a e^2\right) x \sqrt{a-c x^4}}{24 d^2 \left(c d^2-a e^2\right)^2 \left(d+e x^2\right)^2}-$$

$$\frac{e^2 \left(29 c^2 d^4-14 a c d^2 e^2+5 a^2 e^4\right) x \sqrt{a-c x^4}}{16 d^3 \left(c d^2-a e^2\right)^3 \left(d+e x^2\right)}-$$

$$\left(a^{3/4} c^{1/4} e \left(29 c^2 d^4-14 a c d^2 e^2+5 a^2 e^4\right) \sqrt{1-\frac{c x^4}{a}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]\right) \Big/$$

$$\left(16 d^3 \left(c d^2-a e^2\right)^3 \sqrt{a-c x^4}\right)-$$

$$\left(a^{1/4} c^{1/4} \left(57 c^2 d^4-30 \sqrt{a} c^{3/2} d^3 e-32 a c d^2 e^2+10 a^{3/2} \sqrt{c} d e^3+15 a^2 e^4\right) \sqrt{1-\frac{c x^4}{a}}\right.$$

$$\left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]\right) \Big/ \left(48 d^3 \left(\sqrt{c} d-\sqrt{a} e\right)^2 \left(\sqrt{c} d+\sqrt{a} e\right)^3 \sqrt{a-c x^4}\right)+$$

$$\left(a^{1/4} \left(35 c^3 d^6-7 a c^2 d^4 e^2+17 a^2 c d^2 e^4-5 a^3 e^6\right) \sqrt{1-\frac{c x^4}{a}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\sqrt{a} e}{\sqrt{c} d},\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right],-1\right]\right) \Big/ \left(16 c^{1/4} d^4 \left(c d^2-a e^2\right)^3 \sqrt{a-c x^4}\right)$$

Result (type 4, 458 leaves) :

$$\frac{1}{48\, d^4 \sqrt{a - c\, x^4}} \left(-\left(\left(d\, e^2\, x \left(a - c\, x^4\right) \left(8 \left(c\, d^3 - a\, d\, e^2\right)^2 + 10\, d \left(c\, d^2 - a\, e^2\right) \left(3\, c\, d^2 - a\, e^2\right) \left(d + e\, x^2\right) + 3 \left(29\, c^2\, d^4 - 14\, a\, c\, d^2\, e^2 + 5\, a^2\, e^4\right) \left(d + e\, x^2\right)^2\right)\right) \Big/ \left(\left(c\, d^2 - a\, e^2\right)^3 \left(d + e\, x^2\right)^3\right)\right) - \frac{1}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}} \left(-c\, d^2 + a\, e^2\right)^3}\, i \sqrt{1 - \frac{c\, x^4}{a}} \left(3 \sqrt{a} \sqrt{c}\, d\, e \left(29\, c^2\, d^4 - 14\, a\, c\, d^2\, e^2 + 5\, a^2\, e^4\right) \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right], -1\right] + \sqrt{c}\, d \left(57\, c^{5/2}\, d^5 - 87 \sqrt{a}\, c^2\, d^4\, e - 2\, a\, c^{3/2}\, d^3\, e^2 + 42\, a^{3/2}\, c\, d^2\, e^3 + 5\, a^2 \sqrt{c}\, d\, e^4 - 15\, a^{5/2}\, e^5\right) \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right], -1\right] + 3 \left(-35\, c^3\, d^6 + 7\, a\, c^2\, d^4\, e^2 - 17\, a^2\, c\, d^2\, e^4 + 5\, a^3\, e^6\right) \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right], -1\right]\right)\right)$$

Problem 164: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d + e\, x^2}{\sqrt{-a + c\, x^4}}\, dx$$

Optimal (type 4, 126 leaves, 6 steps):

$$\frac{a^{3/4}\, e \sqrt{1 - \frac{c\, x^4}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4}\, x}{a^{1/4}}\right], -1\right]}{c^{3/4} \sqrt{-a + c\, x^4}} + \frac{a^{3/4} \left(\frac{\sqrt{c}\, d}{\sqrt{a}} - e\right) \sqrt{1 - \frac{c\, x^4}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4}\, x}{a^{1/4}}\right], -1\right]}{c^{3/4} \sqrt{-a + c\, x^4}}$$

Result (type 4, 128 leaves):

$$\left(i \sqrt{1-\frac{c x^4}{a}} \left(\sqrt{a}\, e\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right], -1\right] + \left(\sqrt{c}\, d-\sqrt{a}\, e\right) \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right], -1\right]\right)\right) \Big/ \left(\sqrt{a} \left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{-a+c x^4}\right)$$

Problem 165: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right) \sqrt{-a+c x^4}} \, dx$$

Optimal (type 4, 73 leaves, 2 steps):

$$\frac{a^{1/4} \sqrt{1-\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, \text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right], -1\right]}{c^{1/4} d \sqrt{-a+c x^4}}$$

Result (type 4, 92 leaves):

$$-\frac{i \sqrt{1-\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{\sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right], -1\right]}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, d \sqrt{-a+c x^4}}$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a}+\sqrt{c}\, x^2}{\sqrt{-a+c x^4}} \, dx$$

Optimal (type 4, 54 leaves, 3 steps):

$$\frac{a^{3/4} \sqrt{1-\frac{c x^4}{a}}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{c^{1/4} x}{a^{1/4}}\right], -1\right]}{c^{1/4} \sqrt{-a+c x^4}}$$

Result (type 4, 78 leaves):

$$\frac{i \sqrt{c} \sqrt{1-\frac{c x^4}{a}}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right], -1\right]}{\left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{-a+c x^4}}$$

Problem 167: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{\frac{c}{a}}\, x^2}{\sqrt{-a+c\, x^4}}\, dx$$

Optimal (type 4, 52 leaves, 3 steps):

$$\frac{\sqrt{1-\frac{c\, x^4}{a}}\ \text{EllipticE}\left[\text{ArcSin}\left[\left(\frac{c}{a}\right)^{1/4} x\right],\, -1\right]}{\left(\frac{c}{a}\right)^{1/4} \sqrt{-a+c\, x^4}}$$

Result (type 4, 142 leaves):

$$\left(i \sqrt{1-\frac{c\, x^4}{a}} \left(\sqrt{a} \sqrt{\frac{c}{a}}\ \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],\, -1\right] + \left(\sqrt{c}-\sqrt{a} \sqrt{\frac{c}{a}}\right) \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\, x\right],\, -1\right]\right)\right) \Big/ \left(\sqrt{a} \left(-\frac{\sqrt{c}}{\sqrt{a}}\right)^{3/2} \sqrt{-a+c\, x^4}\right)$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e\, x^2}{\sqrt{-a-c\, x^4}}\, dx$$

Optimal (type 4, 236 leaves, 3 steps):

$$-\frac{e\, x \sqrt{-a-c\, x^4}}{\sqrt{c} \left(\sqrt{a}+\sqrt{c}\, x^2\right)} - \frac{a^{1/4} e \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c\, x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticE}\left[2\, \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\, \frac{1}{2}\right]}{c^{3/4} \sqrt{-a-c\, x^4}} + \frac{1}{2\, c^{3/4} \sqrt{-a-c\, x^4}} a^{1/4} \left(\frac{\sqrt{c}\, d}{\sqrt{a}}+e\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c\, x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\ \text{EllipticF}\left[2\, \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\, \frac{1}{2}\right]$$

Result (type 4, 134 leaves):

$$\left(\sqrt{1+\frac{c x^4}{a}}\left(\sqrt{a}\, e\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]+\left(-i \sqrt{c}\, d-\sqrt{a}\, e\right) \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]\right)\right) \Big/ \left(\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c}\, \sqrt{-a-c x^4}\right)$$

Problem 169: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right) \sqrt{-a-c x^4}}\, dx$$

Optimal (type 4, 347 leaves, 3 steps):

$$\frac{\sqrt{e}\, \text{ArcTan}\left[\frac{\sqrt{-c d^2-a e^2}\, x}{\sqrt{d}\, \sqrt{e}\, \sqrt{-a-c x^4}}\right]}{2 \sqrt{d}\, \sqrt{-c d^2-a e^2}}+\frac{c^{1/4}\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{1/4}\left(\sqrt{c}\, d-\sqrt{a}\, e\right) \sqrt{-a-c x^4}}-$$

$$\left(a^{3/4}\left(\frac{\sqrt{c}\, d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, \text{EllipticPi}\left[-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2}{4 \sqrt{a}\, \sqrt{c}\, d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(4 c^{1/4} d\left(c d^2-a e^2\right) \sqrt{-a-c x^4}\right)$$

Result (type 4, 98 leaves):

$$-\frac{i \sqrt{1+\frac{c x^4}{a}}\, \text{EllipticPi}\left[-\frac{i \sqrt{a}\, e}{\sqrt{c}\, d}, i\, \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, x\right],-1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, d \sqrt{-a-c x^4}}$$

Problem 171: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right) \sqrt{4+5 x^4}}\, dx$$

Optimal (type 4, 310 leaves, 3 steps):

$$\frac{\sqrt{b}\ \text{ArcTan}\left[\frac{\sqrt{5 a^2+4 b^2}\ x}{\sqrt{a}\ \sqrt{b}\ \sqrt{4+5 x^4}}\right]}{2 \sqrt{a}\ \sqrt{5 a^2+4 b^2}}+$$

$$\left(5^{1/4} \left(\sqrt{5}\ a+2 b\right) \left(2+\sqrt{5}\ x^2\right) \sqrt{\frac{4+5 x^4}{\left(2+\sqrt{5}\ x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{5^{1/4} x}{\sqrt{2}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(2 \sqrt{2}\ \left(5 a^2-4 b^2\right) \sqrt{4+5 x^4}\right)-$$

$$\left(\left(\sqrt{5}\ a+2 b\right)^2 \left(2+\sqrt{5}\ x^2\right) \sqrt{\frac{4+5 x^4}{\left(2+\sqrt{5}\ x^2\right)^2}}\ \text{EllipticPi}\left[-\frac{\left(\sqrt{5}\ a-2 b\right)^2}{8 \sqrt{5}\ a b}, 2 \text{ArcTan}\left[\frac{5^{1/4} x}{\sqrt{2}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(4 \sqrt{2}\ 5^{1/4} a \left(5 a^2-4 b^2\right) \sqrt{4+5 x^4}\right)$$

Result (type 4, 50 leaves):

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right) \text{EllipticPi}\left[-\frac{2 i b}{\sqrt{5}\ a}, i \text{ArcSinh}\left[\left(\frac{1}{2}+\frac{i}{2}\right) 5^{1/4} x\right], -1\right]}{5^{1/4} a}$$

Problem 172: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right) \sqrt{4-d x^4}}\, dx$$

Optimal (type 4, 40 leaves, 1 step):

$$\frac{\text{EllipticPi}\left[-\frac{2 b}{a \sqrt{d}}, \text{ArcSin}\left[\frac{d^{1/4} x}{\sqrt{2}}\right], -1\right]}{\sqrt{2}\ a d^{1/4}}$$

Result (type 4, 59 leaves):

$$-\frac{i \text{EllipticPi}\left[-\frac{2 b}{a \sqrt{d}}, i \text{ArcSinh}\left[\frac{\sqrt{-\sqrt{d}}\ x}{\sqrt{2}}\right], -1\right]}{\sqrt{2}\ a \sqrt{-\sqrt{d}}}$$

Problem 173: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+b x^2\right) \sqrt{4+d x^4}}\, dx$$

Optimal (type 4, 300 leaves, 3 steps):

$$\frac{\sqrt{b}\,\text{ArcTan}\left[\frac{\sqrt{4b^2+a^2 d}\,x}{\sqrt{a}\,\sqrt{b}\,\sqrt{4+d x^4}}\right]}{2\sqrt{a}\,\sqrt{4b^2+a^2 d}}-\frac{d^{1/4}\left(2+\sqrt{d}\,x^2\right)\sqrt{\frac{4+d x^4}{\left(2+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{d^{1/4}x}{\sqrt{2}}\right],\frac{1}{2}\right]}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+d x^4}}+$$

$$\left(\left(2b+a\sqrt{d}\right)\left(2+\sqrt{d}\,x^2\right)\sqrt{\frac{4+d x^4}{\left(2+\sqrt{d}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}},2\,\text{ArcTan}\left[\frac{d^{1/4}x}{\sqrt{2}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(4\sqrt{2}\,a\left(2b-a\sqrt{d}\right)d^{1/4}\sqrt{4+d x^4}\right)$$

Result (type 4, 65 leaves) :

$$-\frac{i\,\text{EllipticPi}\left[-\frac{2ib}{a\sqrt{d}},i\,\text{ArcSinh}\left[\frac{\sqrt{i\sqrt{d}}\,x}{\sqrt{2}}\right],-1\right]}{\sqrt{2}\,a\sqrt{i\sqrt{d}}}$$

Problem 174: Unable to integrate problem.

$$\int\frac{\sqrt{a+b x^2}}{\sqrt{1-x^4}}\,dx$$

Optimal (type 4, 112 leaves, ? steps):

$$\frac{a\sqrt{1-x^2}\sqrt{\frac{a\left(1+x^2\right)}{a+b x^2}}\,\text{EllipticPi}\left[\frac{b}{a+b},\text{ArcSin}\left[\frac{\sqrt{a+b}\,x}{\sqrt{a+b x^2}}\right],-\frac{a-b}{a+b}\right]}{\sqrt{a+b}\,\sqrt{1+x^2}\sqrt{\frac{a\left(1-x^2\right)}{a+b x^2}}}$$

Result (type 8, 25 leaves) :

$$\int\frac{\sqrt{a+b x^2}}{\sqrt{1-x^4}}\,dx$$

Problem 180: Unable to integrate problem.

$$\int\frac{\left(a+b x^4\right)^p}{c+e x^2}\,dx$$

Optimal (type 6, 123 leaves, 6 steps) :

$$\frac{x\left(a+b x^4\right)^p\left(1+\frac{b x^4}{a}\right)^{-p}\text{AppellF1}\left[\frac{1}{4},-p,1,\frac{5}{4},-\frac{b x^4}{a},\frac{e^2 x^4}{c^2}\right]}{c}-$$

$$\frac{e x^3\left(a+b x^4\right)^p\left(1+\frac{b x^4}{a}\right)^{-p}\text{AppellF1}\left[\frac{3}{4},-p,1,\frac{7}{4},-\frac{b x^4}{a},\frac{e^2 x^4}{c^2}\right]}{3c^2}$$

Result (type 8, 21 leaves) :

$$\int \frac{\left(a+b\,x^4\right)^p}{c+e\,x^2}\,dx$$

Problem 181: Unable to integrate problem.

$$\int \frac{\left(a+b\,x^4\right)^p}{\left(c+e\,x^2\right)^2}\,dx$$

Optimal (type 6, 189 leaves, 8 steps):

$$\frac{x\left(a+b\,x^4\right)^p\left(1+\frac{b\,x^4}{a}\right)^{-p}\text{AppellF1}\left[\frac{1}{4},-p,2,\frac{5}{4},-\frac{b\,x^4}{a},\frac{e^2\,x^4}{c^2}\right]}{c^2}-\frac{2\,e\,x^3\left(a+b\,x^4\right)^p\left(1+\frac{b\,x^4}{a}\right)^{-p}\text{AppellF1}\left[\frac{3}{4},-p,2,\frac{7}{4},-\frac{b\,x^4}{a},\frac{e^2\,x^4}{c^2}\right]}{3\,c^3}+\frac{e^2\,x^5\left(a+b\,x^4\right)^p\left(1+\frac{b\,x^4}{a}\right)^{-p}\text{AppellF1}\left[\frac{5}{4},-p,2,\frac{9}{4},-\frac{b\,x^4}{a},\frac{e^2\,x^4}{c^2}\right]}{5\,c^4}$$

Result (type 8, 21 leaves):

$$\int \frac{\left(a+b\,x^4\right)^p}{\left(c+e\,x^2\right)^2}\,dx$$

Problem 186: Unable to integrate problem.

$$\int \frac{\left(1+b\,x^4\right)^p}{1-x^2}\,dx$$

Optimal (type 6, 50 leaves, 4 steps):

$$x\,\text{AppellF1}\left[\frac{1}{4},1,-p,\frac{5}{4},x^4,-b\,x^4\right]+\frac{1}{3}x^3\,\text{AppellF1}\left[\frac{3}{4},1,-p,\frac{7}{4},x^4,-b\,x^4\right]$$

Result (type 8, 21 leaves):

$$\int \frac{\left(1+b\,x^4\right)^p}{1-x^2}\,dx$$

Problem 187: Unable to integrate problem.

$$\int \frac{\left(1+b\,x^4\right)^p}{\left(1-x^2\right)^2}\,dx$$

Optimal (type 6, 77 leaves, 5 steps):

$$x\,\text{AppellF1}\left[\frac{1}{4},2,-p,\frac{5}{4},x^4,-b\,x^4\right]+\frac{2}{3}x^3\,\text{AppellF1}\left[\frac{3}{4},2,-p,\frac{7}{4},x^4,-b\,x^4\right]+\frac{1}{5}x^5\,\text{AppellF1}\left[\frac{5}{4},2,-p,\frac{9}{4},x^4,-b\,x^4\right]$$

Result (type 8, 21 leaves):

$$\int \frac{\left(1+b x^4\right)^p}{\left(1-x^2\right)^2} \, dx$$

Problem 188: Unable to integrate problem.

$$\int \frac{\left(1+b x^4\right)^p}{\left(1-x^2\right)^3} \, dx$$

Optimal (type 6, 101 leaves, 6 steps):

$$x \, \text{AppellF1}\left[\frac{1}{4}, 3, -p, \frac{5}{4}, x^4, -b x^4\right] + x^3 \, \text{AppellF1}\left[\frac{3}{4}, 3, -p, \frac{7}{4}, x^4, -b x^4\right] + \frac{3}{5} x^5 \, \text{AppellF1}\left[\frac{5}{4}, 3, -p, \frac{9}{4}, x^4, -b x^4\right] + \frac{1}{7} x^7 \, \text{AppellF1}\left[\frac{7}{4}, 3, -p, \frac{11}{4}, x^4, -b x^4\right]$$

Result (type 8, 21 leaves):

$$\int \frac{\left(1+b x^4\right)^p}{\left(1-x^2\right)^3} \, dx$$

Problem 199: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right)^{5/2}}{\sqrt{a^2-b^2 x^4}} \, dx$$

Optimal (type 3, 153 leaves, 5 steps):

$$-\frac{9 a x \left(a-b x^2\right) \sqrt{a+b x^2}}{8 \sqrt{a^2-b^2 x^4}} - \frac{x \left(a-b x^2\right) \left(a+b x^2\right)^{3/2}}{4 \sqrt{a^2-b^2 x^4}} + \frac{19 a^2 \sqrt{a-b x^2} \sqrt{a+b x^2} \, \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a-b x^2}}\right]}{8 \sqrt{b} \sqrt{a^2-b^2 x^4}}$$

Result (type 3, 98 leaves):

$$-\frac{\left(11 a x + 2 b x^3\right) \sqrt{a^2-b^2 x^4}}{8 \sqrt{a+b x^2}} + \frac{19 \, i \, a^2 \, \text{Log}\left[-2 \, i \sqrt{b} \, x + \frac{2 \sqrt{a^2-b^2 x^4}}{\sqrt{a+b x^2}}\right]}{8 \sqrt{b}}$$

Problem 200: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+b x^2\right)^{3/2}}{\sqrt{a^2-b^2 x^4}} \, dx$$

Optimal (type 3, 110 leaves, 4 steps):

$$-\frac{x \left(a-b x^2\right) \sqrt{a+b x^2}}{2 \sqrt{a^2-b^2 x^4}} + \frac{3 a \sqrt{a-b x^2} \sqrt{a+b x^2} \, \text{ArcTan}\left[\frac{\sqrt{b} x}{\sqrt{a-b x^2}}\right]}{2 \sqrt{b} \sqrt{a^2-b^2 x^4}}$$

Result (type 3, 86 leaves):

$$-\frac{x\sqrt{a^2-b^2 x^4}}{2\sqrt{a+b x^2}}+\frac{3\,\mathrm{i}\,a\,\mathrm{Log}\left[-2\,\mathrm{i}\sqrt{b}\,x+\frac{2\sqrt{a^2-b^2 x^4}}{\sqrt{a+b x^2}}\right]}{2\sqrt{b}}$$

Problem 201: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+b x^2}}{\sqrt{a^2-b^2 x^4}}\,dx$$

Optimal (type 3, 65 leaves, 3 steps):

$$\frac{\sqrt{a-b x^2}\,\sqrt{a+b x^2}\,\mathrm{ArcTan}\left[\frac{\sqrt{b}\,x}{\sqrt{a-b x^2}}\right]}{\sqrt{b}\,\sqrt{a^2-b^2 x^4}}$$

Result (type 3, 50 leaves):

$$\frac{\mathrm{i}\,\mathrm{Log}\left[-2\,\mathrm{i}\sqrt{b}\,x+\frac{2\sqrt{a^2-b^2 x^4}}{\sqrt{a+b x^2}}\right]}{\sqrt{b}}$$

Problem 225: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx$$

Optimal (type 4, 183 leaves, 6 steps):

$$\frac{26\,x\sqrt{1+x^2+x^4}}{45\left(1+x^2\right)}+\frac{2}{45}\,x\left(7+6 x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{3}\,x\left(1+x^2+x^4\right)^{3/2}+\frac{1}{9}\,x^3\left(1+x^2+x^4\right)^{3/2}-$$
$$\frac{26\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\mathrm{EllipticE}\left[2\,\mathrm{ArcTan}[x],\frac{1}{4}\right]}{45\sqrt{1+x^2+x^4}}+\frac{7\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,\mathrm{EllipticF}\left[2\,\mathrm{ArcTan}[x],\frac{1}{4}\right]}{15\sqrt{1+x^2+x^4}}$$

Result (type 4, 169 leaves):

$$\frac{1}{45\sqrt{1+x^2+x^4}}$$
$$\left(x\left(29+61 x^2+81 x^4+57 x^6+25 x^8+5 x^{10}\right)+26\,(-1)^{1/3}\sqrt{1+(-1)^{1/3} x^2}\,\sqrt{1-(-1)^{2/3} x^2}\right.$$
$$\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]+2\,(-1)^{5/6}\left(9\,\mathrm{i}+4\sqrt{3}\right)$$
$$\left.\sqrt{1+(-1)^{1/3} x^2}\,\sqrt{1-(-1)^{2/3} x^2}\,\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]\right)$$

Problem 226: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}\, dx$$

Optimal (type 4, 164 leaves, 5 steps):

$$\frac{2 x \sqrt{1+x^2+x^4}}{3 \left(1+x^2\right)}+\frac{2}{21} x \left(4+3 x^2\right) \sqrt{1+x^2+x^4}+\frac{1}{7} x \left(1+x^2+x^4\right)^{3/2}-\frac{2 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{3 \sqrt{1+x^2+x^4}}+\frac{4 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{7 \sqrt{1+x^2+x^4}}$$

Result (type 4, 162 leaves):

$$\frac{1}{21 \sqrt{1+x^2+x^4}}\left(x \left(11+20 x^2+23 x^4+12 x^6+3 x^8\right)+14 (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\, \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]+2 (-1)^{1/3} \left(-7+5 (-1)^{1/3}\right) \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\, \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]\right)$$

Problem 227: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(1+x^2\right) \sqrt{1+x^2+x^4}\, dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$\frac{3 x \sqrt{1+x^2+x^4}}{5 \left(1+x^2\right)}+\frac{1}{5} x \left(2+x^2\right) \sqrt{1+x^2+x^4}-\frac{3 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{5 \sqrt{1+x^2+x^4}}+\frac{3 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{5 \sqrt{1+x^2+x^4}}$$

Result (type 4, 168 leaves):

$$\frac{1}{5 \sqrt{1+x^2+x^4}}\left(2 x+3 x^3+3 x^5+x^7+3 (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\, \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]+\frac{3}{2} \sqrt{2+\left(1-i \sqrt{3}\right) x^2} \sqrt{2+\left(1+i \sqrt{3}\right) x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2} \left(x+i \sqrt{3} x\right)\right],\frac{1}{2} i \left(i+\sqrt{3}\right)\right]\right)$$

Problem 228: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\, dx$$

Optimal (type 4, 137 leaves, 8 steps):

$$\frac{x\sqrt{1+x^2+x^4}}{1+x^2}+\frac{1}{2}\,\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 118 leaves):

$$-\frac{1}{\sqrt{1+x^2+x^4}}(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\;\sqrt{1-(-1)^{2/3}x^2}\;\left(-\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+(-1)^{1/3}\,\text{EllipticPi}\left[(-1)^{1/3},-\text{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 229: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^2}\, dx$$

Optimal (type 4, 49 leaves, 1 step):

$$\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{2\sqrt{1+x^2+x^4}}$$

Result (type 4, 164 leaves):

$$\frac{1}{2\sqrt{1+x^2+x^4}}\left(\frac{x+x^3+x^5}{1+x^2}+(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\;\sqrt{1-(-1)^{2/3}x^2}\;\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\;\sqrt{1-(-1)^{2/3}x^2}\;\left(-\text{EllipticE}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)\right)$$

Problem 230: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^3}\,dx$$

Optimal (type 4, 93 leaves, 23 steps):

$$\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{1}{4}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 176 leaves):

$$\frac{1}{4\sqrt{1+x^2+x^4}}\left(\frac{x\left(2+x^2\right)\left(1+x^2+x^4\right)}{\left(1+x^2\right)^2}+(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\right.$$
$$\left(-\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)-$$
$$2\,(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}$$
$$\left.\text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 231: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^4}\,dx$$

Optimal (type 4, 166 leaves, 26 steps):

$$\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\left(1+x^2\right)^2}+\frac{1}{4}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+$$
$$\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{8\sqrt{1+x^2+x^4}}$$

Result (type 4, 240 leaves):

$$\frac{1}{6\sqrt{1+x^2+x^4}}\left(\frac{x\left(1+x^2+x^4\right)\left(4+5x^2+2x^4\right)}{\left(1+x^2\right)^3}-2\,(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\right.$$
$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)-$$
$$(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-3\,(-1)^{2/3}$$
$$\left.\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 232: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1+x^2\right)^3}{\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 159 leaves, 5 steps):

$$\frac{11}{15} x \sqrt{1+x^2+x^4}+\frac{1}{5} x^3 \sqrt{1+x^2+x^4}+\frac{14 x \sqrt{1+x^2+x^4}}{15 \left(1+x^2\right)}-\frac{14 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{15 \sqrt{1+x^2+x^4}}+\frac{3 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{5 \sqrt{1+x^2+x^4}}$$

Result (type 4, 157 leaves):

$$\frac{1}{15 \sqrt{1+x^2+x^4}}\left(x \left(11+14 x^2+14 x^4+3 x^6\right)+14 (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\ \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]+2 (-1)^{1/3} \left(-7+2 (-1)^{1/3}\right) \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\ \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]\right)$$

Problem 233: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(1+x^2\right)^2}{\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 137 leaves, 4 steps):

$$\frac{1}{3} x \sqrt{1+x^2+x^4}+\frac{4 x \sqrt{1+x^2+x^4}}{3 \left(1+x^2\right)}-\frac{4 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{3 \sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}[x],\frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}$$

Result (type 4, 143 leaves):

$$\frac{1}{3 \sqrt{1+x^2+x^4}}\left(x+x^3+x^5+4 (-1)^{1/3} \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\ \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]+2 (-1)^{1/3} \left(-2+(-1)^{1/3}\right) \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2}\ \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right],(-1)^{2/3}\right]\right)$$

Problem 234: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\, dx$$

Optimal (type 4, 115 leaves, 3 steps):

$$\frac{x\sqrt{1+x^2+x^4}}{1+x^2}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}$$

Result (type 4, 94 leaves):

$$\frac{1}{\sqrt{1+x^2+x^4}}(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\left(-1+(-1)^{1/3}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 235: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\, dx$$

Optimal (type 4, 69 leaves, 4 steps):

$$\frac{1}{2}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 73 leaves):

$$-\frac{1}{\sqrt{1+x^2+x^4}}(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\ \sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]$$

Problem 236: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\, dx$$

Optimal (type 4, 118 leaves, 8 steps):

$$\frac{1}{2}\,\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{2\sqrt{1+x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 226 leaves):

$$\frac{1}{2\sqrt{1+x^2+x^4}}\left(\frac{x+x^3+x^5}{1+x^2}-(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\left(-\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)-2\,(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\;\text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 237: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{1}{4}\,\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}-\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{2\sqrt{1+x^2+x^4}}$$

Result (type 4, 235 leaves):

$$\frac{1}{4\sqrt{1+x^2+x^4}}\left(\frac{x\left(4+3x^2\right)\left(1+x^2+x^4\right)}{\left(1+x^2\right)^2}-3\,(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)-2\,(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-2\,(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\;\text{EllipticPi}\left[(-1)^{1/3},-i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 238: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}} \, dx$$

Optimal (type 4, 144 leaves, 4 steps):

$$-\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)}-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{\sqrt{1+x^2+x^4}}$$

Result (type 4, 136 leaves):

$$\frac{1}{3\sqrt{1+x^2+x^4}}\left(-x+x^3+2(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+2(-1)^{5/6}\sqrt{3+3(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 239: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}} \, dx$$

Optimal (type 4, 98 leaves, 2 steps):

$$\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}-\frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)}+\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}$$

Result (type 4, 158 leaves):

$$\frac{1}{3\sqrt{1+x^2+x^4}}\left(x+2x^3-2(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-i\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\sqrt{6+\left(3-3i\sqrt{3}\right)x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\left(x+i\sqrt{3}\,x\right)\right],\frac{1}{2}i\left(i+\sqrt{3}\right)\right]\right)$$

Problem 240: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 96 leaves, 2 steps):

$$\frac{x\left(2+x^2\right)}{3\sqrt{1+x^2+x^4}}-\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}$$

Result (type 4, 160 leaves):

$$\frac{1}{3\sqrt{1+x^2+x^4}}\left(2x+x^3-(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-\frac{1}{2}\mathrm{i}\sqrt{2+\left(1+\mathrm{i}\sqrt{3}\right)x^2}\,\sqrt{6+\left(3-3\,\mathrm{i}\sqrt{3}\right)x^2}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{2}\left(x+\mathrm{i}\sqrt{3}\,x\right)\right],\frac{1}{2}\mathrm{i}\left(\mathrm{i}+\sqrt{3}\right)\right]\right)$$

Problem 241: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(1+x^2\right)\left(1+x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 166 leaves, 9 steps):

$$-\frac{x\left(1+2x^2\right)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\frac{1}{2}\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]-\frac{2\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticE}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{3\sqrt{1+x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\ \text{EllipticF}\left[2\,\text{ArcTan}[x],\frac{1}{4}\right]}{4\sqrt{1+x^2+x^4}}$$

Result (type 4, 204 leaves):

$$\frac{1}{3\sqrt{1+x^2+x^4}}\left(-x-2x^3+2(-1)^{1/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]+(-1)^{1/3}\left(-2+(-1)^{1/3}\right)\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]-3(-1)^{2/3}\sqrt{1+(-1)^{1/3}x^2}\,\sqrt{1-(-1)^{2/3}x^2}\ \text{EllipticPi}\left[(-1)^{1/3},-\mathrm{i}\,\text{ArcSinh}\left[(-1)^{5/6}x\right],(-1)^{2/3}\right]\right)$$

Problem 242: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(1+x^2\right)^2 \left(1+x^2+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 111 leaves, 16 steps):

$$-\frac{x \left(2+x^2\right)}{3 \sqrt{1+x^2+x^4}}+\frac{x \sqrt{1+x^2+x^4}}{3 \left(1+x^2\right)}+\text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{\left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{6 \sqrt{1+x^2+x^4}}$$

Result (type 4, 168 leaves):

$$\left(-2 x \left(1+x^2\right) \left(2+x^2\right)+3 x \left(1+x^2+x^4\right)-(-1)^{1/3} \left(1+x^2\right) \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]+\left(-1+5 (-1)^{1/3}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]+12 (-1)^{1/3} \text{EllipticPi}\left[(-1)^{1/3}, -i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]\right)\right) \Big/ \left(6 \left(1+x^2\right) \sqrt{1+x^2+x^4}\right)$$

Problem 243: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(1+x^2\right)^3 \left(1+x^2+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 190 leaves, 23 steps):

$$-\frac{x \left(1-x^2\right)}{3 \sqrt{1+x^2+x^4}}+\frac{x \sqrt{1+x^2+x^4}}{4 \left(1+x^2\right)^2}-\frac{x \sqrt{1+x^2+x^4}}{3 \left(1+x^2\right)}+\frac{3}{4} \text{ArcTan}\left[\frac{x}{\sqrt{1+x^2+x^4}}\right]+\frac{19 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{12 \sqrt{1+x^2+x^4}}-\frac{5 \left(1+x^2\right) \sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}[x], \frac{1}{4}\right]}{4 \sqrt{1+x^2+x^4}}$$

Result (type 4, 192 leaves):

$$\frac{1}{12 \left(1+x^2\right)^2 \sqrt{1+x^2+x^4}} \left(4 x \left(-1+x^2\right) \left(1+x^2\right)^2+3 x \left(1+x^2+x^4\right)+15 x \left(1+x^2\right) \left(1+x^2+x^4\right)-(-1)^{1/3} \left(1+x^2\right)^2 \sqrt{1+(-1)^{1/3} x^2} \sqrt{1-(-1)^{2/3} x^2} \left(19 \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]+\left(-9+10 i \sqrt{3}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]+18 (-1)^{1/3} \text{EllipticPi}\left[(-1)^{1/3}, -i \text{ArcSinh}\left[(-1)^{5/6} x\right], (-1)^{2/3}\right]\right)\right)$$

Problem 286: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^3 \sqrt{2+3 x^2+x^4}\, dx$$

Optimal (type 4, 193 leaves, 6 steps):

$$\frac{577 x \left(2+x^2\right)}{3 \sqrt{2+3 x^2+x^4}}+\frac{1}{21} x \left(2608+757 x^2\right) \sqrt{2+3 x^2+x^4}+\frac{275}{7} x \left(2+3 x^2+x^4\right)^{3/2}+\frac{125}{9} x^3 \left(2+3 x^2+x^4\right)^{3/2}-\frac{577 \sqrt{2} \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\, \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3 \sqrt{2+3 x^2+x^4}}+\frac{2945 \sqrt{2} \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\, \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{21 \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 119 leaves):

$$\frac{1}{63 \sqrt{2+3 x^2+x^4}}\left(25548 x+61214 x^3+57312 x^5+28496 x^7+7725 x^9+875 x^{11}-12117 i \sqrt{1+x^2} \sqrt{2+x^2}\, \text{EllipticE}\left[i\, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-5553 i \sqrt{1+x^2} \sqrt{2+x^2}\, \text{EllipticF}\left[i\, \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 287: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^2 \sqrt{2+3 x^2+x^4}\, dx$$

Optimal (type 4, 168 leaves, 5 steps):

$$\frac{31 x \left(2+x^2\right)}{\sqrt{2+3 x^2+x^4}}+\frac{1}{21} x \left(407+114 x^2\right) \sqrt{2+3 x^2+x^4}+\frac{25}{7} x \left(2+3 x^2+x^4\right)^{3/2}-\frac{31 \sqrt{2} \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\, \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3 x^2+x^4}}+\frac{472 \sqrt{2} \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\, \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{21 \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 114 leaves):

$$\frac{1}{21\sqrt{2+3x^2+x^4}}\left(1114x+2349x^3+1724x^5+564x^7+75x^9-651\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-293\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 288: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)\sqrt{2+3x^2+x^4}\,dx$$

Optimal (type 4, 149 leaves, 4 steps):

$$\frac{5x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\left(10+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{5\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{11\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}$$

Result (type 4, 109 leaves):

$$\frac{1}{3\sqrt{2+3x^2+x^4}}\left(20x+36x^3+19x^5+3x^7-15\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-7\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 289: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{2+3x^2+x^4}\,dx$$

Optimal (type 4, 141 leaves, 4 steps):

$$\frac{x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\sqrt{2+3x^2+x^4}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{2\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}$$

Result (type 4, 102 leaves):

$$\frac{1}{3\sqrt{2+3x^2+x^4}}\left(2x+3x^3+x^5-3\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 290: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$$

Optimal (type 4, 178 leaves, 8 steps):

$$\frac{x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{2+3x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{35\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 90 leaves):

$$-\left(\left(i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\left(35\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+21\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-6\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\right)\Big/\left(175\sqrt{2+3x^2+x^4}\right)\right)$$

Problem 291: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx$$

Optimal (type 4, 209 leaves, 8 steps):

$$-\frac{x\left(2+x^2\right)}{70\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{14\left(7+5x^2\right)}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{35\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{140\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\left(2+x^2\right)\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{980\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 208 leaves):

$$\left(350 x+525 x^3+175 x^5+35 i \sqrt{1+x^2} \sqrt{2+x^2} \left(7+5 x^2\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-84 i \sqrt{1+x^2} \sqrt{2+x^2} \left(7+5 x^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-7 i \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticPi}\left[\frac{10}{7},i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-5 i x^2 \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticPi}\left[\frac{10}{7},i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(2450 \left(7+5 x^2\right) \sqrt{2+3 x^2+x^4}\right)$$

Problem 292: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+3 x^2+x^4}}{\left(7+5 x^2\right)^3} \, dx$$

Optimal (type 4, 237 leaves, 25 steps):

$$-\frac{11 x \left(2+x^2\right)}{11760 \sqrt{2+3 x^2+x^4}}+\frac{x \sqrt{2+3 x^2+x^4}}{28 \left(7+5 x^2\right)^2}+\frac{11 x \sqrt{2+3 x^2+x^4}}{2352 \left(7+5 x^2\right)}+\frac{11 \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5880 \sqrt{2} \sqrt{2+3 x^2+x^4}}+\frac{81 \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}} \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{7840 \sqrt{2} \sqrt{2+3 x^2+x^4}}-\frac{1201 \left(2+x^2\right) \text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{164640 \sqrt{2} \sqrt{\frac{2+x^2}{1+x^2}} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 174 leaves):

$$\left(\frac{14700 x \left(2+3 x^2+x^4\right)}{\left(7+5 x^2\right)^2}+\frac{1925 x \left(2+3 x^2+x^4\right)}{7+5 x^2}+385 i \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-434 i \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-1201 i \sqrt{1+x^2} \sqrt{2+x^2} \text{EllipticPi}\left[\frac{10}{7},i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(411600 \sqrt{2+3 x^2+x^4}\right)$$

Problem 293: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^3 \left(2+3 x^2+x^4\right)^{3/2} \, dx$$

Optimal (type 4, 219 leaves, 7 steps):

$$\frac{20884\,x\,(2+x^2)}{65\sqrt{2+3x^2+x^4}}+\frac{x\,(1032541+297911\,x^2)\sqrt{2+3x^2+x^4}}{5005}+\frac{x\,(208212+65345\,x^2)\,(2+3x^2+x^4)^{3/2}}{3003}+\frac{3825}{143}\,x\,(2+3x^2+x^4)^{5/2}+\frac{125}{13}\,x^3\,(2+3x^2+x^4)^{5/2}-\frac{20884\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{65\sqrt{2+3x^2+x^4}}+\frac{1171349\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5005\sqrt{2+3x^2+x^4}}$$

Result (type 4, 129 leaves) :

$$\left(13572486\,x+40493455\,x^3+54938052\,x^5+46218643\,x^7+25350660\,x^9+8705725\,x^{11}+1701000\,x^{13}+144375\,x^{15}-4824204\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-2203890\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(15015\sqrt{2+3x^2+x^4}\right)$$

Problem 294: Result unnecessarily involves imaginary or complex numbers.

$$\int (7+5x^2)^2\,(2+3x^2+x^4)^{3/2}\,dx$$

Optimal (type 4, 198 leaves, 6 steps) :

$$\frac{742\,x\,(2+x^2)}{15\sqrt{2+3x^2+x^4}}+\frac{x\,(36783+10643\,x^2)\sqrt{2+3x^2+x^4}}{1155}+\frac{1}{693}\,x\,(7281+2240\,x^2)\,(2+3x^2+x^4)^{3/2}+\frac{25}{11}\,x\,(2+3x^2+x^4)^{5/2}-\frac{742\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{15\sqrt{2+3x^2+x^4}}+\frac{13879\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{385\sqrt{2+3x^2+x^4}}$$

Result (type 4, 124 leaves) :

$$\left(429318\,x+1160065\,x^3+1333551\,x^5+892084\,x^7+363480\,x^9+82075\,x^{11}+7875\,x^{13}-171402\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-78420\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(3465\,\sqrt{2+3x^2+x^4}\right)$$

Problem 295: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}\,dx$$

Optimal (type 4, 179 leaves, 5 steps):

$$\frac{116\,x\left(2+x^2\right)}{15\sqrt{2+3x^2+x^4}}+\frac{1}{105}\,x\left(519+149x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{63}\,x\left(108+35x^2\right)\left(2+3x^2+x^4\right)^{3/2}-\frac{116\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{15\sqrt{2+3x^2+x^4}}+\frac{197\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{35\sqrt{2+3x^2+x^4}}$$

Result (type 4, 119 leaves):

$$\left(5274\,x+12745\,x^3+12018\,x^5+5962\,x^7+1590\,x^9+175\,x^{11}-2436\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-1110\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(315\,\sqrt{2+3x^2+x^4}\right)$$

Problem 296: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(2+3x^2+x^4\right)^{3/2}\,dx$$

Optimal (type 4, 172 leaves, 5 steps):

$$\frac{6\,x\left(2+x^2\right)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{35}\,x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}\,x\left(2+3x^2+x^4\right)^{3/2}-\frac{6\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{2+3x^2+x^4}}+\frac{31\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{35\sqrt{2+3x^2+x^4}}$$

Result (type 4, 114 leaves):

$$\frac{1}{35\sqrt{2+3x^2+x^4}}\left(78x+165x^3+121x^5+39x^7+5x^9-42\,i\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-20\,i\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 297: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3x^2+x^4\right)^{3/2}}{7+5x^2}\,dx$$

Optimal (type 4, 207 leaves, 13 steps):

$$\frac{24x\left(2+x^2\right)}{125\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{24\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{125\sqrt{2+3x^2+x^4}}+\frac{56\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{375\sqrt{2+3x^2+x^4}}-\frac{9\sqrt{2}\left(2+x^2\right)\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{875\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3x^2+x^4}}$$

Result (type 4, 148 leaves):

$$\left(3850x+6825x^3+3500x^5+525x^7-2520\,i\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-1022\,i\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-108\,i\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(13125\sqrt{2+3x^2+x^4}\right)$$

Problem 298: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx$$

Optimal (type 4, 222 leaves, 21 steps):

$$\frac{9 x \left(2+x^2\right)}{175 \sqrt{2+3 x^2+x^4}}+\frac{1}{75} x \sqrt{2+3 x^2+x^4}-\frac{3 x \sqrt{2+3 x^2+x^4}}{175 \left(7+5 x^2\right)}-\frac{9 \sqrt{2} \left(1+x^2\right) \sqrt{\frac{2+x^2}{1+x^2}}\ \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{175 \sqrt{2+3 x^2+x^4}}+\frac{59 \left(1+x^2\right) \sqrt{\frac{2+x^2}{2+2 x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{1050 \sqrt{2+3 x^2+x^4}}+\frac{9 \left(1+x^2\right) \sqrt{\frac{2+x^2}{2+2 x^2}}\ \text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{2450 \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 213 leaves):

$$\frac{1}{18375 \left(7+5 x^2\right) \sqrt{2+3 x^2+x^4}}\left(2800 x+6650 x^3+5075 x^5+1225 x^7-945 i \sqrt{1+x^2} \sqrt{2+x^2} \left(7+5 x^2\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-182 i \sqrt{1+x^2} \sqrt{2+x^2} \left(7+5 x^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+189 i \sqrt{1+x^2} \sqrt{2+x^2}\ \text{EllipticPi}\left[\frac{10}{7},i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+135 i x^2 \sqrt{1+x^2} \sqrt{2+x^2}\ \text{EllipticPi}\left[\frac{10}{7},i \text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 299: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(2+3 x^2+x^4\right)^{3/2}}{\left(7+5 x^2\right)^3}\, dx$$

Optimal (type 4, 231 leaves, 27 steps):

$$\frac{3 x \left(2+x^2\right)}{392 \sqrt{2+3 x^2+x^4}}-\frac{3 x \sqrt{2+3 x^2+x^4}}{350 \left(7+5 x^2\right)^2}+\frac{17 x \sqrt{2+3 x^2+x^4}}{9800 \left(7+5 x^2\right)}-\frac{3 \left(1+x^2\right) \sqrt{\frac{2+x^2}{2+2 x^2}}\ \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{196 \sqrt{2+3 x^2+x^4}}+\frac{5 \left(1+x^2\right) \sqrt{\frac{2+x^2}{2+2 x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{784 \sqrt{2+3 x^2+x^4}}+\frac{141 \left(2+x^2\right) \text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{27440 \sqrt{2} \sqrt{\frac{2+x^2}{1+x^2}} \sqrt{2+3 x^2+x^4}}$$

Result (type 4, 174 leaves):

$$\left(-\frac{588\,x\,\left(2+3\,x^2+x^4\right)}{\left(7+5\,x^2\right)^2}+\frac{119\,x\,\left(2+3\,x^2+x^4\right)}{7+5\,x^2}-525\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-406\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+141\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(68\,600\,\sqrt{2+3\,x^2+x^4}\right)$$

Problem 300: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5\,x^2\right)^3}{\sqrt{2+3\,x^2+x^4}}\,dx$$

Optimal (type 4, 157 leaves, 5 steps):

$$\frac{135\,x\,\left(2+x^2\right)}{\sqrt{2+3\,x^2+x^4}}+75\,x\,\sqrt{2+3\,x^2+x^4}+25\,x^3\,\sqrt{2+3\,x^2+x^4}-\frac{135\,\sqrt{2}\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3\,x^2+x^4}}+\frac{193\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 106 leaves):

$$\frac{1}{\sqrt{2+3\,x^2+x^4}}\left(25\,x\,\left(6+11\,x^2+6\,x^4+x^6\right)-135\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-58\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 301: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5\,x^2\right)^2}{\sqrt{2+3\,x^2+x^4}}\,dx$$

Optimal (type 4, 142 leaves, 4 steps):

$$\frac{20\,x\,\left(2+x^2\right)}{\sqrt{2+3\,x^2+x^4}}+\frac{25}{3}\,x\,\sqrt{2+3\,x^2+x^4}-\frac{20\,\sqrt{2}\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3\,x^2+x^4}}+\frac{97\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\,\sqrt{2}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 104 leaves):

$$\frac{1}{3\sqrt{2+3x^2+x^4}}\left(25\,x\,\left(2+3x^2+x^4\right)-60\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-37\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 302: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 121 leaves, 3 steps):

$$\frac{5x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}-\frac{5\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{7\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 69 leaves):

$$-\frac{1}{\sqrt{2+3x^2+x^4}}\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\left(5\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 303: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 48 leaves, 1 step):

$$\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 50 leaves):

$$-\frac{i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]}{\sqrt{2+3x^2+x^4}}$$

Problem 304: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 106 leaves, 4 steps):

$$\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{2\sqrt{2}\ \sqrt{2+3x^2+x^4}}-\frac{5\left(2+x^2\right)\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{14\sqrt{2}\ \sqrt{\frac{2+x^2}{1+x^2}}\ \sqrt{2+3x^2+x^4}}$$

Result (type 4, 55 leaves):

$$-\frac{i\sqrt{1+x^2}\ \sqrt{2+x^2}\ \text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]}{7\sqrt{2+3x^2+x^4}}$$

Problem 305: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 209 leaves, 9 steps):

$$\frac{5x\left(2+x^2\right)}{84\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)}-\frac{5\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\ \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{42\sqrt{2}\ \sqrt{2+3x^2+x^4}}+$$
$$\frac{9\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{56\sqrt{2}\ \sqrt{2+3x^2+x^4}}-\frac{65\left(2+x^2\right)\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{1176\sqrt{2}\ \sqrt{\frac{2+x^2}{1+x^2}}\ \sqrt{2+3x^2+x^4}}$$

Result (type 4, 208 leaves):

$$\left(-350x-525x^3-175x^5-35i\sqrt{1+x^2}\ \sqrt{2+x^2}\ \left(7+5x^2\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-\right.$$
$$14i\sqrt{1+x^2}\ \sqrt{2+x^2}\ \left(7+5x^2\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-$$
$$91i\sqrt{1+x^2}\ \sqrt{2+x^2}\ \text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-$$
$$\left.65i\,x^2\sqrt{1+x^2}\ \sqrt{2+x^2}\ \text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(588\left(7+5x^2\right)\sqrt{2+3x^2+x^4}\right)$$

Problem 306: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 237 leaves, 10 steps):

$$\frac{65\,x\,(2+x^2)}{4704\sqrt{2+3x^2+x^4}}-\frac{25\,x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2}-$$
$$\frac{325\,x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)}-\frac{65\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{2352\sqrt{2}\sqrt{2+3x^2+x^4}}+$$
$$\frac{631\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{9408\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{2525\,(2+x^2)\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{65856\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 186 leaves):

$$\Big(-175\,x\,(238+487x^2+314x^4+65x^6)-$$
$$455\,i\sqrt{1+x^2}\sqrt{2+x^2}\,(7+5x^2)^2\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+$$
$$14\,i\sqrt{1+x^2}\sqrt{2+x^2}\,(7+5x^2)^2\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-$$
$$505\,i\sqrt{1+x^2}\sqrt{2+x^2}\,(7+5x^2)^2\,\text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\Big)\Big/$$
$$\left(32928\,(7+5x^2)^2\sqrt{2+3x^2+x^4}\right)$$

Problem 307: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx$$

Optimal (type 4, 189 leaves, 6 steps):

$$\frac{7679\,x\,(2+x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{x\,(115+179x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{5000}{3}\,x\sqrt{2+3x^2+x^4}+625\,x^3\sqrt{2+3x^2+x^4}-$$
$$\frac{7679\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{15383\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 109 leaves):

$$\frac{1}{6\sqrt{2+3x^2+x^4}}\left(19655\,x+36963\,x^3+21250\,x^5+3750\,x^7-23037\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-7729\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 308: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^4}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 170 leaves, 5 steps):

$$\frac{637\,x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(145+113x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{625}{3}x\sqrt{2+3x^2+x^4}-\frac{637\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{1067\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}$$

Result (type 4, 104 leaves):

$$\frac{1}{6\sqrt{2+3x^2+x^4}}\left(2935\,x+4089\,x^3+1250\,x^5-1911\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-2357\,i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 309: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^3}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 149 leaves, 4 steps):

$$\frac{x\left(5-11x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{261\,x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}-\frac{261\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{169\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 99 leaves):

$$-\frac{1}{2\sqrt{2+3x^2+x^4}}\left(-5x+11x^3+261\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+77\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 310: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 149 leaves, 4 steps):

$$-\frac{17x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(25+17x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{17\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{6\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}$$

Result (type 4, 99 leaves):

$$\frac{1}{2\sqrt{2+3x^2+x^4}}\left(25x+17x^3+17\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-41\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 311: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$-\frac{x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(5+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 97 leaves):

$$\frac{1}{2\sqrt{2+3x^2+x^4}}\left(5x+x^3+i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-3i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 312: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 149 leaves, 4 steps):

$$-\frac{3x\left(2+x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{x\left(5+3x^2\right)}{2\sqrt{2+3x^2+x^4}}+\frac{3\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}$$

Result (type 4, 99 leaves):

$$\frac{1}{2\sqrt{2+3x^2+x^4}}\left(5x+3x^3+3i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 313: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 173 leaves, 9 steps):

$$\frac{x}{6\sqrt{2+3x^2+x^4}}+\frac{\sqrt{2}\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2+3x^2+x^4}}-\frac{9\left(1+x^2\right)\sqrt{\frac{2+x^2}{2+2x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{4\sqrt{2+3x^2+x^4}}+\frac{125\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{84\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 138 leaves):

$$\frac{1}{42\sqrt{2+3x^2+x^4}}\left(35x+14x^3+14\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-7\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+25\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 314: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^2\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 235 leaves, 19 steps):

$$-\frac{31x\left(2+x^2\right)}{56\sqrt{2+3x^2+x^4}}+\frac{x\left(20+11x^2\right)}{36\sqrt{2+3x^2+x^4}}+\frac{625x\sqrt{2+3x^2+x^4}}{504\left(7+5x^2\right)}+\frac{31\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{28\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{463\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{336\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{375\left(2+x^2\right)\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{784\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 208 leaves):

$$\frac{1}{1176\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\left(7490x+10157x^3+3255x^5+651\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\left(7+5x^2\right)\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+182\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\left(7+5x^2\right)\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+1575\,\mathrm{i}\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+1125\,\mathrm{i}\,x^2\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{10}{7},\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 315: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^3\left(2+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 263 leaves, 29 steps):

$$-\frac{5797\,x\,(2+x^2)}{28\,224\,\sqrt{2+3x^2+x^4}}+\frac{x\,(50+23x^2)}{216\,\sqrt{2+3x^2+x^4}}+\frac{625\,x\,\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2}+$$
$$\frac{41\,875\,x\,\sqrt{2+3x^2+x^4}}{84\,672\,(7+5x^2)}+\frac{5797\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\ \text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{14\,112\,\sqrt{2}\,\sqrt{2+3x^2+x^4}}-$$
$$\frac{49\,907\,(1+x^2)\,\sqrt{\frac{2+x^2}{1+x^2}}\ \text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{56\,448\,\sqrt{2}\,\sqrt{2+3x^2+x^4}}+\frac{192\,625\,(2+x^2)\,\text{EllipticPi}\left[\frac{2}{7},\text{ArcTan}[x],\frac{1}{2}\right]}{395\,136\,\sqrt{2}\,\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3x^2+x^4}}$$

Result (type 4, 159 leaves):

$$\left(\frac{7\,x\,(550\,550+1\,089\,803\,x^2+698\,290\,x^4+144\,925\,x^6)}{(7+5x^2)^2}+\right.$$
$$40\,579\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\ \text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-$$
$$742\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\ \text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+$$
$$\left.38\,525\,i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\ \text{EllipticPi}\left[\frac{10}{7},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\Big/\left(197\,568\,\sqrt{2+3x^2+x^4}\right)$$

Problem 316: Result unnecessarily involves imaginary or complex numbers.

$$\int (7+5x^2)^4\,\sqrt{2+x^2-x^4}\,dx$$

Optimal (type 4, 116 leaves, 8 steps):

$$\frac{1}{231}\,x\,(177\,953+717\,372\,x^2)\,\sqrt{2+x^2-x^4}-$$
$$\frac{116\,100}{77}\,x\,(2+x^2-x^4)^{3/2}-\frac{14\,500}{33}\,x^3\,(2+x^2-x^4)^{3/2}-\frac{625}{11}\,x^5\,(2+x^2-x^4)^{3/2}+$$
$$\frac{3\,764\,813}{231}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{539\,419}{77}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 112 leaves):

$$\frac{1}{231\,\sqrt{2+x^2-x^4}}\left(-1\,037\,294\,x-186\,503\,x^3+1\,125\,819\,x^5+231\,228\,x^7-105\,925\,x^9-\right.$$
$$75\,250\,x^{11}-13\,125\,x^{13}+3\,764\,813\,i\,\sqrt{4+2x^2-2x^4}\ \text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-$$
$$\left.4\,838\,091\,i\,\sqrt{4+2x^2-2x^4}\ \text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 317: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^3 \sqrt{2+x^2-x^4}\, dx$$

Optimal (type 4, 95 leaves, 7 steps):

$$\frac{1}{63} x \left(5956+14691 x^2\right) \sqrt{2+x^2-x^4}-\frac{1825}{21} x \left(2+x^2-x^4\right)^{3/2}-\frac{125}{9} x^3 \left(2+x^2-x^4\right)^{3/2}+\frac{79411}{63} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{8735}{21} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 107 leaves):

$$\frac{1}{63 \sqrt{2+x^2-x^4}}\left(-9988 x+9938 x^3+21660 x^5-1116 x^7-3725 x^9-875 x^{11}+79411 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-106014 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 318: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4}\, dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$\frac{1}{21} x \left(275+354 x^2\right) \sqrt{2+x^2-x^4}-\frac{25}{7} x \left(2+x^2-x^4\right)^{3/2}+\frac{2045}{21} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{79}{7} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 102 leaves):

$$\frac{1}{21 \sqrt{2+x^2-x^4}}\left(250 x+683 x^3+304 x^5-204 x^7-75 x^9+2045 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-2949 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 319: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \left(7+5 x^2\right) \sqrt{2+x^2-x^4}\, dx$$

Optimal (type 4, 46 leaves, 5 steps):

$$x\left(2+x^2\right)\sqrt{2+x^2-x^4}+7\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+3\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 94 leaves):

$$\frac{1}{\sqrt{2+x^2-x^4}}\left(4x+4x^3-x^5-x^7+7\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-12\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 320: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\sqrt{2+x^2-x^4}\,dx$$

Optimal (type 4, 44 leaves, 5 steps):

$$\frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 90 leaves):

$$\frac{1}{3\sqrt{2+x^2-x^4}}\left(2x+x^3-x^5+i\sqrt{4+2x^2-2x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-3\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 321: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx$$

Optimal (type 4, 46 leaves, 7 steps):

$$-\frac{1}{5}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{17}{25}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{34}{175}\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 51 leaves):

$$-\frac{1}{175}\,i\sqrt{2}\left(35\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+7\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-17\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 322: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5 x^2\right)^2} \, dx$$

Optimal (type 4, 74 leaves, 7 steps):

$$\frac{x \sqrt{2+x^2-x^4}}{14 \left(7+5 x^2\right)}+\frac{1}{70} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{6}{175} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{99 \text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{2450}$$

Result (type 4, 196 leaves):

$$\left(700 x+350 x^3-350 x^5+70 i \sqrt{2} \left(7+5 x^2\right) \sqrt{2+x^2-x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-21 i \sqrt{2} \left(7+5 x^2\right) \sqrt{2+x^2-x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-693 i \sqrt{2} \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]-495 i \sqrt{2} x^2 \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)\Big/\left(4900 \left(7+5 x^2\right) \sqrt{2+x^2-x^4}\right)$$

Problem 323: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2+x^2-x^4}}{\left(7+5 x^2\right)^3} \, dx$$

Optimal (type 4, 102 leaves, 21 steps):

$$\frac{x \sqrt{2+x^2-x^4}}{28 \left(7+5 x^2\right)^2}-\frac{31 x \sqrt{2+x^2-x^4}}{13328 \left(7+5 x^2\right)}-\frac{31 \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{66640}-\frac{269 \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{166600}+\frac{16601 \text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{2332400}$$

Result (type 4, 244 leaves):

$$\frac{1}{4664800 \left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4}}\left(181300 x-17850 x^3-144900 x^5+54250 x^7-2170 i \sqrt{2} \left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]+7021 i \sqrt{2} \left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-813449 i \sqrt{2} \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]-1162070 i \sqrt{2} x^2 \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]-415025 i \sqrt{2} x^4 \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 324: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^4 \left(2+x^2-x^4\right)^{3/2} dx$$

Optimal (type 4, 142 leaves, 9 steps):

$$\frac{3 x \left(2193559+7837383 x^2\right) \sqrt{2+x^2-x^4}}{5005}-\frac{x \left(69817-1581440 x^2\right) \left(2+x^2-x^4\right)^{3/2}}{1001}-\frac{132300}{143} x \left(2+x^2-x^4\right)^{5/2}-\frac{11750}{39} x^3 \left(2+x^2-x^4\right)^{5/2}-\frac{125}{3} x^5 \left(2+x^2-x^4\right)^{5/2}+\frac{124141422 \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{5005}-\frac{50794416 \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{5005}$$

Result (type 4, 122 leaves):

$$\left(-75836958 x+48624305 x^3+172881581 x^5+32834763 x^7-36649955 x^9-24642275 x^{11}-1556625 x^{13}+2646875 x^{15}+625625 x^{17}+372424266 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-482444775 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)\Big/\left(15015 \sqrt{2+x^2-x^4}\right)$$

Problem 325: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(7+5 x^2\right)^3 \left(2+x^2-x^4\right)^{3/2} dx$$

Optimal (type 4, 121 leaves, 8 steps):

$$\frac{x\left(2512273+5712051 x^2\right)\sqrt{2+x^2-x^4}}{15015}+\frac{x\left(33792+374045 x^2\right)\left(2+x^2-x^4\right)^{3/2}}{3003}-\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{125}{13}x^3\left(2+x^2-x^4\right)^{5/2}+\frac{31072528\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{15015}-\frac{3199778\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{5005}$$

Result (type 4, 117 leaves):

$$\left(-872614 x+11078615 x^3+13371048 x^5-1756521 x^7-4448240 x^9-1027775 x^{11}+388500 x^{13}+144375 x^{15}+31072528\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-41809125\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)\Big/\left(15015\sqrt{2+x^2-x^4}\right)$$

Problem 326: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^2\left(2+x^2-x^4\right)^{3/2}dx$$

Optimal (type 4, 100 leaves, 7 steps):

$$\frac{1}{495}x\left(11497+14889x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{99}x\left(363+920x^2\right)\left(2+x^2-x^4\right)^{3/2}-\frac{25}{11}x\left(2+x^2-x^4\right)^{5/2}+\frac{85942}{495}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{3392}{165}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 112 leaves):

$$\frac{1}{495\sqrt{2+x^2-x^4}}\left(21254x+53435x^3+23097x^5-19944x^7-10760x^9+1225x^{11}+1125x^{13}+85942\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-123825\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 327: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}dx$$

Optimal (type 4, 81 leaves, 6 steps):

$$\frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}+\frac{4432}{315}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{418}{105}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 107 leaves):

$$\frac{1}{315\sqrt{2+x^2-x^4}}\left(3134\,x+4085\,x^3-438\,x^5-1674\,x^7-110\,x^9+175\,x^{11}+4432\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-7275\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 328: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(2+x^2-x^4\right)^{3/2}dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$\frac{1}{35}x\left(19+3\,x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{34}{35}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{48}{35}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 102 leaves):

$$\frac{1}{35\sqrt{2+x^2-x^4}}\left(58\,x+45\,x^3-31\,x^5-13\,x^7+5\,x^9+34\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-75\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 329: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(2+x^2-x^4\right)^{3/2}}{7+5\,x^2}dx$$

Optimal (type 4, 72 leaves, 13 steps):

$$\frac{1}{75}x\left(13-3\,x^2\right)\sqrt{2+x^2-x^4}+\frac{92}{375}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{178}{625}\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{1156\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{4375}$$

Result (type 4, 130 leaves):

$$\left(4550\,x+1225\,x^3-2800\,x^5+525\,x^7+3220\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-2961\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-1734\,i\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)\Big/\left(13125\sqrt{2+x^2-x^4}\right)$$

Problem 330: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5 x^2\right)^2} \, dx$$

Optimal (type 4, 93 leaves, 21 steps):

$$-\frac{1}{75} x \sqrt{2+x^2-x^4}-\frac{17 x \sqrt{2+x^2-x^4}}{175 \left(7+5 x^2\right)}-\frac{97}{525} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{458}{875} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{1241 \text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{6125}$$

Result (type 4, 201 leaves):

$$\frac{1}{36750 \left(7+5 x^2\right) \sqrt{2+x^2-x^4}}\left(-14000 x-11900 x^3+4550 x^5+2450 x^7-6790 i \sqrt{2} \left(7+5 x^2\right) \sqrt{2+x^2-x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]+567 i \sqrt{2} \left(7+5 x^2\right) \sqrt{2+x^2-x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]+26061 i \sqrt{2} \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]+18615 i \sqrt{2} x^2 \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 331: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5 x^2\right)^3} \, dx$$

Optimal (type 4, 102 leaves, 27 steps):

$$-\frac{17 x \sqrt{2+x^2-x^4}}{350 \left(7+5 x^2\right)^2}+\frac{563 x \sqrt{2+x^2-x^4}}{9800 \left(7+5 x^2\right)}+\frac{191 \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{9800}-\frac{1251 \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{24500}+\frac{9879 \text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{343000}$$

Result (type 4, 244 leaves):

$$\frac{1}{686000\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}}\left(485100\,x+636650\,x^3-45500\,x^5-197050\,x^7+13370\,i\sqrt{2}\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-2541\,i\sqrt{2}\left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-484071\,i\sqrt{2}\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-691530\,i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-246975\,i\sqrt{2}\,x^4\sqrt{2+x^2-x^4}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 332: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^3}{\sqrt{2+x^2-x^4}}\,dx$$

Optimal (type 4, 65 leaves, 6 steps):

$$-\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{3905}{3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-542\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 97 leaves):

$$\frac{1}{6\sqrt{2+x^2-x^4}}\left(-2500\,x-1550\,x^3+1100\,x^5+150\,x^7+7810\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-10089\,i\sqrt{4+2x^2-2x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 333: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^2}{\sqrt{2+x^2-x^4}}\,dx$$

Optimal (type 4, 46 leaves, 5 steps):

$$-\frac{25}{3}x\sqrt{2+x^2-x^4}+\frac{260}{3}\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-21\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 92 leaves):

$$\frac{1}{6\sqrt{2+x^2-x^4}}\left(-100\,x-50\,x^3+50\,x^5+520\,\mathrm{i}\,\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]-717\,\mathrm{i}\,\sqrt{4+2\,x^2-2\,x^4}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 334: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{7+5\,x^2}{\sqrt{2+x^2-x^4}}\,dx$$

Optimal (type 4, 25 leaves, 4 steps):

$$5\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+2\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 34 leaves):

$$\frac{1}{\sqrt{2}}\mathrm{i}\left(10\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]-17\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 335: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{2+x^2-x^4}}\,dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 19 leaves):

$$-\frac{\mathrm{i}\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]}{\sqrt{2}}$$

Problem 336: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5\,x^2\right)\sqrt{2+x^2-x^4}}\,dx$$

Optimal (type 4, 17 leaves, 2 steps):

$$\frac{1}{7}\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 24 leaves):

$$-\frac{\mathrm{i}\,\text{EllipticPi}\left[\frac{5}{7},\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]}{7\sqrt{2}}$$

Problem 337: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4}} \, dx$$

Optimal (type 4, 74 leaves, 8 steps):

$$-\frac{25 x \sqrt{2+x^2-x^4}}{476 \left(7+5 x^2\right)}-\frac{5}{476} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{1}{238} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{167 \text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{3332}$$

Result (type 4, 196 leaves):

$$\left(-700 x-350 x^3+350 x^5-70 i \sqrt{2} \left(7+5 x^2\right) \sqrt{2+x^2-x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]+119 i \sqrt{2} \left(7+5 x^2\right) \sqrt{2+x^2-x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-1169 i \sqrt{2} \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]-835 i \sqrt{2} x^2 \sqrt{2+x^2-x^4} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)\Big/\left(6664 \left(7+5 x^2\right) \sqrt{2+x^2-x^4}\right)$$

Problem 338: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5 x^2\right)^3 \sqrt{2+x^2-x^4}} \, dx$$

Optimal (type 4, 102 leaves, 9 steps):

$$-\frac{25 x \sqrt{2+x^2-x^4}}{952 \left(7+5 x^2\right)^2}-\frac{12525 x \sqrt{2+x^2-x^4}}{453152 \left(7+5 x^2\right)}-\frac{2505 \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{453152}-\frac{263 \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{226576}+\frac{58915 \text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{3172064}$$

Result (type 4, 108 leaves):

$$\frac{1}{6344128}\left(\frac{350 x \left(-7966-8993 x^2+1478 x^4+2505 x^6\right)}{\left(7+5 x^2\right)^2 \sqrt{2+x^2-x^4}}-35070 i \sqrt{2} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]+56287 i \sqrt{2} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]-58915 i \sqrt{2} \text{EllipticPi}\left[\frac{5}{7},i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 339: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^5}{\left(2+x^2-x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 93 leaves, 7 steps):

$$\frac{x \left(1419985+1419793 x^2\right)}{18 \sqrt{2+x^2-x^4}}+\frac{27500}{3} x \sqrt{2+x^2-x^4}+625 x^3 \sqrt{2+x^2-x^4}-\frac{3482293}{18} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{627857}{6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 97 leaves):

$$\frac{1}{18 \sqrt{2+x^2-x^4}} \left(1749985 x+1607293 x^3-153750 x^5-11250 x^7-3482293 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]+4281654 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 340: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^4}{\left(2+x^2-x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 74 leaves, 6 steps):

$$\frac{x \left(83585+83489 x^2\right)}{18 \sqrt{2+x^2-x^4}}+\frac{625}{3} x \sqrt{2+x^2-x^4}-\frac{165239}{18} \text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{31921}{6} \text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 92 leaves):

$$\frac{1}{18 \sqrt{2+x^2-x^4}} \left(91085 x+87239 x^3-3750 x^5-165239 i \sqrt{4+2 x^2-2 x^4} \text{EllipticE}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]+199977 i \sqrt{4+2 x^2-2 x^4} \text{EllipticF}\left[i \text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 341: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^3}{\left(2+x^2-x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 55 leaves, 5 steps):

$$\frac{x\left(4945+4897 x^2\right)}{18 \sqrt{2+x^2-x^4}}-\frac{7147}{18}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{1763}{6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 79 leaves):

$$\frac{1}{18}\left(\frac{4945 x}{\sqrt{2+x^2-x^4}}+\frac{4897 x^3}{\sqrt{2+x^2-x^4}}-7147\, i \sqrt{2}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+8076\, i \sqrt{2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 342: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 55 leaves, 5 steps):

$$\frac{x\left(305+281 x^2\right)}{18 \sqrt{2+x^2-x^4}}-\frac{281}{18}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{139}{6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 79 leaves):

$$\frac{1}{18}\left(\frac{305 x}{\sqrt{2+x^2-x^4}}+\frac{281 x^3}{\sqrt{2+x^2-x^4}}-281\, i \sqrt{2}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+213\, i \sqrt{2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 343: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{7+5 x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 55 leaves, 5 steps):

$$\frac{x\left(25+13 x^2\right)}{18 \sqrt{2+x^2-x^4}}-\frac{13}{18}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{17}{6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 79 leaves):

$$\frac{1}{18}\left(\frac{25 x}{\sqrt{2+x^2-x^4}}+\frac{13 x^3}{\sqrt{2+x^2-x^4}}-13\, i \sqrt{2}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-6\, i \sqrt{2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 344: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 55 leaves, 5 steps):

$$\frac{x\left(5-x^2\right)}{18\sqrt{2+x^2-x^4}}+\frac{1}{18}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{1}{6}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 79 leaves):

$$\frac{1}{18}\left(\frac{5\,x}{\sqrt{2+x^2-x^4}}-\frac{x^3}{\sqrt{2+x^2-x^4}}+i\sqrt{2}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-3\,i\sqrt{2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 345: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 72 leaves, 8 steps):

$$\frac{x\left(35-16x^2\right)}{306\sqrt{2+x^2-x^4}}+\frac{8}{153}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]+\frac{1}{102}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]-\frac{25}{238}\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]$$

Result (type 4, 101 leaves):

$$\frac{1}{4284}\left(\frac{490\,x}{\sqrt{2+x^2-x^4}}-\frac{224\,x^3}{\sqrt{2+x^2-x^4}}+224\,i\sqrt{2}\,\text{EllipticE}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]-357\,i\sqrt{2}\,\text{EllipticF}\left[i\,\text{ArcSinh}[x],-\frac{1}{2}\right]+225\,i\sqrt{2}\,\text{EllipticPi}\left[\frac{5}{7},i\,\text{ArcSinh}[x],-\frac{1}{2}\right]\right)$$

Problem 346: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^2\left(2+x^2-x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 100 leaves, 17 steps):

$$\frac{x\left(580-287x^2\right)}{10404\sqrt{2+x^2-x^4}}+\frac{625\,x\sqrt{2+x^2-x^4}}{16184\left(7+5x^2\right)}+\frac{5143\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{145656}+\frac{89\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{24276}-\frac{10825\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{113288}$$

Result (type 4, 196 leaves):

$$\frac{1}{2039184\,(7+5x^2)\sqrt{2+x^2-x^4}}\Bigg(953260\,x+253386\,x^3-360010\,x^5+72002\,\mathrm{i}\sqrt{2}\,(7+5x^2)\sqrt{2+x^2-x^4}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]-111741\,\mathrm{i}\sqrt{2}\,(7+5x^2)\sqrt{2+x^2-x^4}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]+681975\,\mathrm{i}\sqrt{2}\sqrt{2+x^2-x^4}\;\text{EllipticPi}\left[\frac{5}{7},\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]+487125\,\mathrm{i}\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\;\text{EllipticPi}\left[\frac{5}{7},\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]\Bigg)$$

Problem 347: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(7+5x^2)^3\,(2+x^2-x^4)^{3/2}}\,dx$$

Optimal (type 4, 128 leaves, 26 steps):

$$\frac{x\,(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}}+\frac{625\,x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2}+\frac{645625\,x\sqrt{2+x^2-x^4}}{15407168\,(7+5x^2)}+\frac{3086453\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{138664512}+\frac{60409\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{23110752}-\frac{6898575\,\text{EllipticPi}\left[-\frac{10}{7},\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right],-2\right]}{107850176}$$

Result (type 4, 244 leaves):

$$\frac{1}{1941303168\,(7+5x^2)^2\sqrt{2+x^2-x^4}}\Bigg(3857257460\,x+3876617542\,x^3-737347940\,x^5-1080258550\,x^7+43210342\,\mathrm{i}\sqrt{2}\,(7+5x^2)^2\sqrt{2+x^2-x^4}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]-67352691\,\mathrm{i}\sqrt{2}\,(7+5x^2)^2\sqrt{2+x^2-x^4}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]+3042271575\,\mathrm{i}\sqrt{2}\sqrt{2+x^2-x^4}\;\text{EllipticPi}\left[\frac{5}{7},\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]+4346102250\,\mathrm{i}\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\;\text{EllipticPi}\left[\frac{5}{7},\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]+1552179375\,\mathrm{i}\sqrt{2}\,x^4\sqrt{2+x^2-x^4}\;\text{EllipticPi}\left[\frac{5}{7},\mathrm{i}\,\text{ArcSinh}[x],-\frac{1}{2}\right]\Bigg)$$

Problem 348: Result unnecessarily involves imaginary or complex numbers.

$$\int(7+5x^2)^4\sqrt{4+3x^2+x^4}\,dx$$

Optimal (type 4, 242 leaves, 7 steps):

$$\frac{51665\,x\sqrt{4+3x^2+x^4}}{33\,(2+x^2)}+\frac{1}{33}x\,(18727+4516x^2)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\,(4+3x^2+x^4)^{3/2}+\frac{23500}{99}x^3\,(4+3x^2+x^4)^{3/2}+\frac{625}{11}x^5\,(4+3x^2+x^4)^{3/2}-\frac{51665\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{33\sqrt{4+3x^2+x^4}}+\frac{33159\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{11\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 354 leaves):

$$\frac{1}{396\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\,(663924+1257535x^2+1217475x^4+712748x^6+264075x^8+57250x^{10}+5625x^{12})-154995\sqrt{2}\,(3i+\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+3\sqrt{2}\,(-36253i+51665\sqrt{7})\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 349: Result unnecessarily involves imaginary or complex numbers.

$$\int (7+5x^2)^3\sqrt{4+3x^2+x^4}\,dx$$

Optimal (type 4, 221 leaves, 6 steps):

$$\frac{4717\,x\,\sqrt{4+3\,x^2+x^4}}{21\,\left(2+x^2\right)}+\frac{1}{21}\,x\,\left(1708+407\,x^2\right)\,\sqrt{4+3\,x^2+x^4}+\frac{275}{7}\,x\,\left(4+3\,x^2+x^4\right)^{3/2}+\frac{125}{9}\,x^3\,\left(4+3\,x^2+x^4\right)^{3/2}-\frac{4717\,\sqrt{2}\,\left(2+x^2\right)\,\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{21\,\sqrt{4+3\,x^2+x^4}}+\frac{1301\,\left(2+x^2\right)\,\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3\,\sqrt{2}\,\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 349 leaves):

$$\left(4\,\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\,x\,\left(60096+93656\,x^2+71862\,x^4+30946\,x^6+7725\,x^8+875\,x^{10}\right)-14151\,\sqrt{2}\,\left(3\,i+\sqrt{7}\right)\,\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+3\,\sqrt{2}\,\left(-3409\,i+4717\,\sqrt{7}\right)\,\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\Big/\left(252\,\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\,\sqrt{4+3\,x^2+x^4}\right)$$

Problem 350: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5\,x^2\right)^2\,\sqrt{4+3\,x^2+x^4}\,dx$$

Optimal (type 4, 198 leaves, 5 steps):

$$\frac{319\,x\,\sqrt{4+3\,x^2+x^4}}{7\,\left(2+x^2\right)}+\frac{1}{7}\,x\,\left(119+38\,x^2\right)\,\sqrt{4+3\,x^2+x^4}+\frac{25}{7}\,x\,\left(4+3\,x^2+x^4\right)^{3/2}-\frac{319\,\sqrt{2}\,\left(2+x^2\right)\,\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{7\,\sqrt{4+3\,x^2+x^4}}+\frac{81\,\left(2+x^2\right)\,\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{2}\,\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 343 leaves):

$$\left(4\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\;x\left(876+1109\,x^2+658\,x^4+188\,x^6+25\,x^8\right)-319\sqrt{2}\left(3\,i+\sqrt{7}\right)\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\;x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+\sqrt{2}\left(-35\,i+319\sqrt{7}\right)\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\;x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\Big/\left(28\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\sqrt{4+3\,x^2+x^4}\right)$$

Problem 351: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5\,x^2\right)\sqrt{4+3\,x^2+x^4}\;dx$$

Optimal (type 4, 177 leaves, 4 steps):

$$\frac{9\,x\sqrt{4+3\,x^2+x^4}}{2+x^2}+\frac{1}{3}\,x\left(10+3\,x^2\right)\sqrt{4+3\,x^2+x^4}-\frac{9\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3\,x^2+x^4}}+\frac{49\left(2+x^2\right)\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3\sqrt{2}\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 338 leaves):

$$\left(4 \sqrt{-\frac{i}{-3 i+\sqrt{7}}}\; x \left(40+42 x^2+19 x^4+3 x^6\right)-27 \sqrt{2} \left(3 i+\sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}}\right.$$

$$\sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}}\; \text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\; x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+$$

$$\sqrt{2} \left(-7 i+27 \sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}}\; \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}}$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\; x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right) \Bigg/ \left(12 \sqrt{-\frac{i}{-3 i+\sqrt{7}}}\; \sqrt{4+3 x^2+x^4}\right)$$

Problem 352: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{4+3 x^2+x^4}\, dx$$

Optimal (type 4, 169 leaves, 4 steps):

$$\frac{1}{3} x \sqrt{4+3 x^2+x^4}+\frac{x \sqrt{4+3 x^2+x^4}}{2+x^2}-\frac{\sqrt{2} \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\; \text{EllipticE}\left[2 \,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3 x^2+x^4}}+$$

$$\frac{7 \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\; \text{EllipticF}\left[2 \,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 331 leaves):

$$\left(4 \sqrt{-\frac{i}{-3 i+\sqrt{7}}}\; x \left(4+3 x^2+x^4\right)-3 \sqrt{2} \left(3 i+\sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}}\right.$$

$$\sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}}\; \text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\; x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+$$

$$\sqrt{2} \left(-7 i+3 \sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}}\; \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}}$$

$$\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\; x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right) \Bigg/ \left(12 \sqrt{-\frac{i}{-3 i+\sqrt{7}}}\; \sqrt{4+3 x^2+x^4}\right)$$

Problem 353: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$$

Optimal (type 4, 322 leaves, 7 steps):

$$\frac{x\sqrt{4+3x^2+x^4}}{5\left(2+x^2\right)}+\frac{1}{5}\sqrt{\frac{11}{35}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]-\frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{5\sqrt{4+3x^2+x^4}}+\frac{9\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{25\sqrt{2}\sqrt{4+3x^2+x^4}}-\frac{11\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{75\sqrt{4+3x^2+x^4}}+\frac{187\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{525\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 283 leaves):

$$-\left(\left(\sqrt{1-\frac{2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{1+\frac{2\,i\,x^2}{3\,i+\sqrt{7}}}\left(35\left(3\,i+\sqrt{7}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+\left(7\,i-35\sqrt{7}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+88\,i\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\right)\Big/\left(350\sqrt{2}\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)\right)$$

Problem 354: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^2}\,dx$$

Optimal (type 4, 284 leaves, 7 steps):

$$-\frac{x\sqrt{4+3x^2+x^4}}{70\left(2+x^2\right)}+\frac{x\sqrt{4+3x^2+x^4}}{14\left(7+5x^2\right)}+\frac{51\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{280\sqrt{385}}+$$
$$\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{35\sqrt{2}\sqrt{4+3x^2+x^4}}-$$
$$\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{35\sqrt{2}\sqrt{4+3x^2+x^4}}+$$
$$\frac{289\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{9800\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 481 leaves):

$$\frac{1}{9800\sqrt{-\frac{i}{-3 i+\sqrt{7}}}\left(7+5 x^2\right)\sqrt{4+3 x^2+x^4}}$$
$$\left(700\sqrt{-\frac{i}{-3 i+\sqrt{7}}}\, x\left(4+3 x^2+x^4\right)+35\left(3 i+\sqrt{7}\right)\left(7+5 x^2\right)\sqrt{2-\frac{4 i x^2}{-3 i+\sqrt{7}}}\right.$$
$$\sqrt{1+\frac{2 i x^2}{3 i+\sqrt{7}}}\left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]-\right.$$
$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right)-98 i\left(7+5 x^2\right)$$
$$\sqrt{2-\frac{4 i x^2}{-3 i+\sqrt{7}}}\sqrt{1+\frac{2 i x^2}{3 i+\sqrt{7}}}\,\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]-$$
$$102 i\left(7+5 x^2\right)\sqrt{2-\frac{4 i x^2}{-3 i+\sqrt{7}}}\sqrt{1+\frac{2 i x^2}{3 i+\sqrt{7}}}$$
$$\left.\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right), i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right)$$

Problem 355: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{4+3 x^2+x^4}}{\left(7+5 x^2\right)^3}\,dx$$

Optimal (type 4, 312 leaves, 18 steps):

$$-\frac{139 x\sqrt{4+3 x^2+x^4}}{86240\left(2+x^2\right)}+\frac{x\sqrt{4+3 x^2+x^4}}{28\left(7+5 x^2\right)^2}+\frac{139 x\sqrt{4+3 x^2+x^4}}{17248\left(7+5 x^2\right)}+$$
$$\frac{14999\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\, x}{\sqrt{4+3 x^2+x^4}}\right]}{344960\sqrt{385}}+\frac{139\left(2+x^2\right)\sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{43120\sqrt{2}\sqrt{4+3 x^2+x^4}}-$$
$$\frac{23\left(2+x^2\right)\sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2940\sqrt{2}\sqrt{4+3 x^2+x^4}}+$$
$$\frac{254983\left(2+x^2\right)\sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{36220800\sqrt{2}\sqrt{4+3 x^2+x^4}}$$

Result (type 4, 308 leaves):

$$\left(\frac{700\,x\,\left(1589+695\,x^2\right)\,\left(4+3\,x^2+x^4\right)}{\left(7+5\,x^2\right)^2}+i\,\sqrt{6+2\,i\,\sqrt{7}}\,\sqrt{1-\frac{2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{1+\frac{2\,i\,x^2}{3\,i+\sqrt{7}}}\right.$$
$$\left(4865\,\left(3-i\,\sqrt{7}\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\,\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+\left(-9597+4865\,i\,\sqrt{7}\right)\right.$$
$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\,\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]-29\,998\,\text{EllipticPi}\left[\frac{5}{14}\,\left(3+i\,\sqrt{7}\right),\right.$$
$$\left.\left.\left.i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\,\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\right)\Big/\left(12\,073\,600\,\sqrt{4+3\,x^2+x^4}\right)$$

Problem 356: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5\,x^2\right)^4\,\left(4+3\,x^2+x^4\right)^{3/2}\,dx$$

Optimal (type 4, 268 leaves, 8 steps):

$$\frac{12\,665\,086\,x\,\sqrt{4+3\,x^2+x^4}}{2145\,\left(2+x^2\right)}+\frac{7\,x\,\left(661\,429+174\,989\,x^2\right)\,\sqrt{4+3\,x^2+x^4}}{2145}+$$
$$\frac{x\,\left(452\,001+131\,080\,x^2\right)\,\left(4+3\,x^2+x^4\right)^{3/2}}{1287}+\frac{92\,150}{429}\,x\,\left(4+3\,x^2+x^4\right)^{5/2}+\frac{2250}{13}\,x^3\,\left(4+3\,x^2+x^4\right)^{5/2}+$$
$$\frac{125}{3}\,x^5\,\left(4+3\,x^2+x^4\right)^{5/2}-\frac{12\,665\,086\,\sqrt{2}\,\left(2+x^2\right)\,\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\,\frac{1}{8}\right]}{2145\,\sqrt{4+3\,x^2+x^4}}+$$
$$\frac{2\,383\,556\,\sqrt{2}\,\left(2+x^2\right)\,\sqrt{\frac{4+3\,x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\,\frac{1}{8}\right]}{429\,\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 364 leaves):

$$\frac{1}{12870\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

$$\left(2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\,\left(180184116+391419623x^2+472235001x^4+377574349x^6+\right.\right.$$
$$\left.212188905x^8+83076275x^{10}+21862875x^{12}+3526875x^{14}+268125x^{16}\right)-$$
$$18997629\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}$$
$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+$$
$$21\sqrt{2}\left(-477617i+904649\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 357: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^3\left(4+3x^2+x^4\right)^{3/2}dx$$

Optimal (type 4, 247 leaves, 7 steps):

$$\frac{4525662\,x\sqrt{4+3x^2+x^4}}{5005\left(2+x^2\right)}+\frac{x\left(1653701+435441x^2\right)\sqrt{4+3x^2+x^4}}{5005}+$$
$$\frac{x\left(53504+15365x^2\right)\left(4+3x^2+x^4\right)^{3/2}}{1001}+\frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2}-$$
$$\frac{4525662\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{5005\sqrt{4+3x^2+x^4}}+$$
$$\frac{121826\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{143\sqrt{4+3x^2+x^4}}$$

Result (type 4, 358 leaves):

$$\frac{1}{10010\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(19463124+36710547x^2+37166164x^4+24107711x^6+10713970x^8+3158575x^{10}+567000x^{12}+48125x^{14}\right)-2262831\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(-1215823i+2262831\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 358: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)^2\left(4+3x^2+x^4\right)^{3/2}dx$$

Optimal (type 4, 226 leaves, 6 steps):

$$\frac{175346\,x\sqrt{4+3x^2+x^4}}{1155\left(2+x^2\right)}+\frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}-\frac{175346\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{1155\sqrt{4+3x^2+x^4}}+\frac{4628\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{33\sqrt{4+3x^2+x^4}}$$

Result (type 4, 354 leaves):

$$\frac{1}{6930\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\right.$$
$$\left(1824876+2932753x^2+2435811x^4+1229714x^6+408480x^8+82075x^{10}+7875x^{12}\right)-$$
$$263019\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}$$
$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+$$
$$3\sqrt{2}\left(-34209i+87673\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 359: Result unnecessarily involves imaginary or complex numbers.

$$\int\left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}dx$$

Optimal (type 4, 207 leaves, 5 steps):

$$\frac{2798x\sqrt{4+3x^2+x^4}}{105\left(2+x^2\right)}+\frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+$$
$$\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}-\frac{2798\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{105\sqrt{4+3x^2+x^4}}+$$
$$\frac{74\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3\sqrt{4+3x^2+x^4}}$$

Result (type 4, 349 leaves):

$$\left(2\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\;x\left(20988+28489\,x^2+19068\,x^4+7082\,x^6+1590\,x^8+175\,x^{10}\right)-\right.$$
$$4197\sqrt{2}\left(3\,i+\sqrt{7}\right)\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}$$
$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\;x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+$$
$$3\sqrt{2}\left(-567\,i+1399\sqrt{7}\right)\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\;x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\bigg/\left(630\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\sqrt{4+3\,x^2+x^4}\right)$$

Problem 360: Result unnecessarily involves imaginary or complex numbers.

$$\int \left(4+3\,x^2+x^4\right)^{3/2}\,dx$$

Optimal (type 4, 198 leaves, 5 steps):

$$\frac{138\,x\sqrt{4+3\,x^2+x^4}}{35\left(2+x^2\right)}+\frac{1}{35}\,x\left(49+9\,x^2\right)\sqrt{4+3\,x^2+x^4}+$$
$$\frac{1}{7}\,x\left(4+3\,x^2+x^4\right)^{3/2}-\frac{138\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{35\sqrt{4+3\,x^2+x^4}}+$$
$$\frac{4\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 343 leaves):

$$\left(2\sqrt{-\frac{i}{-3 i+\sqrt{7}}}\, x\left(276+303 x^2+161 x^4+39 x^6+5 x^8\right)-69\sqrt{2}\left(3 i+\sqrt{7}\right)\sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}}\right.$$

$$\sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+$$

$$\sqrt{2}\left(-77 i+69\sqrt{7}\right)\sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}}\sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right)\Bigg/\left(70\sqrt{-\frac{i}{-3 i+\sqrt{7}}}\sqrt{4+3 x^2+x^4}\right)$$

Problem 361: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(4+3 x^2+x^4\right)^{3/2}}{7+5 x^2}\,dx$$

Optimal (type 4, 284 leaves, 12 steps):

$$\frac{94 x\sqrt{4+3 x^2+x^4}}{125\left(2+x^2\right)}+\frac{1}{75}x\left(11+3 x^2\right)\sqrt{4+3 x^2+x^4}+\frac{44}{125}\sqrt{\frac{11}{35}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3 x^2+x^4}}\right]-$$

$$\frac{94\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{125\sqrt{4+3 x^2+x^4}}+$$

$$\frac{54\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{125\sqrt{4+3 x^2+x^4}}+$$

$$\frac{4114\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{13125\sqrt{4+3 x^2+x^4}}$$

Result (type 4, 477 leaves):

$$\frac{1}{26250\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(350\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(44+45x^2+20x^4+3x^6\right)-4935\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+7\sqrt{2}\left(-241i+705\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-5808i\sqrt{2}\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 362: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx$$

Optimal (type 4, 305 leaves, 19 steps):

$$\frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)}+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{13}{350}\sqrt{\frac{11}{35}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]-\frac{4\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{175\sqrt{4+3x^2+x^4}}+\frac{4\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{175\sqrt{4+3x^2+x^4}}+\frac{2431\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{36750\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 309 leaves):

$$\left(\frac{175\,x\,(23+7\,x^2)\,(4+3\,x^2+x^4)}{7+5\,x^2}-i\,\sqrt{6+2\,i\,\sqrt{7}}\,\sqrt{1-\frac{2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{1+\frac{2\,i\,x^2}{3\,i+\sqrt{7}}}\right.$$
$$\left(105\,(3-i\,\sqrt{7})\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+\right.$$
$$7\,(158+15\,i\,\sqrt{7})\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+429\,\text{EllipticPi}\left[\right.$$
$$\left.\left.\left.\frac{5}{14}\,(3+i\,\sqrt{7}),\,i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\right)\Big/\left(18\,375\,\sqrt{4+3\,x^2+x^4}\right)$$

Problem 363: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(4+3\,x^2+x^4)^{3/2}}{(7+5\,x^2)^3}\,dx$$

Optimal (type 4, 440 leaves, 22 steps):

$$\frac{9\,x\,\sqrt{4+3\,x^2+x^4}}{1960\,(2+x^2)}+\frac{11\,x\,\sqrt{4+3\,x^2+x^4}}{175\,(7+5\,x^2)^2}+\frac{167\,x\,\sqrt{4+3\,x^2+x^4}}{9800\,(7+5\,x^2)}+$$
$$\frac{1347\,\text{ArcTan}\left[\frac{2\,\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3\,x^2+x^4}}\right]}{7840\,\sqrt{385}}+\frac{111\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{24\,500\,\sqrt{2}\,\sqrt{4+3\,x^2+x^4}}-$$
$$\frac{6\,\sqrt{2}\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{875\,\sqrt{4+3\,x^2+x^4}}-$$
$$\frac{817\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{91\,875\,\sqrt{2}\,\sqrt{4+3\,x^2+x^4}}-$$
$$\frac{22\,\sqrt{2}\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{13\,125\,\sqrt{4+3\,x^2+x^4}}+$$
$$\frac{7633\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{274\,400\,\sqrt{2}\,\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 309 leaves):

$$\left(\frac{140\,x\,(357+167\,x^2)\,(4+3\,x^2+x^4)}{(7+5\,x^2)^2}-i\,\sqrt{6+2\,i\,\sqrt{7}}\,\sqrt{1-\frac{2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{1+\frac{2\,i\,x^2}{3\,i+\sqrt{7}}}\left(315\,(3-i\,\sqrt{7})\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+7\,(103+45\,i\,\sqrt{7})\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+2694\,\text{EllipticPi}\left[\frac{5}{14}\,(3+i\,\sqrt{7}),\,i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\right)\Big/\left(274400\,\sqrt{4+3\,x^2+x^4}\right)$$

Problem 364: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(7+5\,x^2)^3}{\sqrt{4+3\,x^2+x^4}}\,dx$$

Optimal (type 4, 187 leaves, 5 steps):

$$75\,x\,\sqrt{4+3\,x^2+x^4}+25\,x^3\,\sqrt{4+3\,x^2+x^4}-\frac{15\,x\,\sqrt{4+3\,x^2+x^4}}{2+x^2}+\frac{15\,\sqrt{2}\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3\,x^2+x^4}}+\frac{13\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2\,\sqrt{2}\,\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 337 leaves):

$$\left(100\,\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\,x\,(12+13\,x^2+6\,x^4+x^6)+15\,\sqrt{2}\,(3\,i+\sqrt{7})\,\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]-\sqrt{2}\,(131\,i+15\,\sqrt{7})\,\sqrt{\frac{-3\,i+\sqrt{7}-2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{\frac{3\,i+\sqrt{7}+2\,i\,x^2}{3\,i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\Big/\left(4\,\sqrt{-\frac{i}{-3\,i+\sqrt{7}}}\,\sqrt{4+3\,x^2+x^4}\right)$$

Problem 365: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5x^2\right)^2}{\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 170 leaves, 4 steps):

$$\frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{20x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{20\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3x^2+x^4}}+\frac{167\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{6\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 331 leaves):

$$\left(50\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(4+3x^2+x^4\right)-30\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(43i+30\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(6\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 366: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 151 leaves, 3 steps):

$$\frac{5x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{5\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{\sqrt{4+3x^2+x^4}}+\frac{17\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 214 leaves):

$$\left(\sqrt{1-\frac{2 i x^2}{-3 i+\sqrt{7}}}\,\sqrt{1+\frac{2 i x^2}{3 i+\sqrt{7}}}\left(-5\left(3 i+\sqrt{7}\right)\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\,x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+\left(i+5\sqrt{7}\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\,x\right],\frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right)\right)\Big/\left(2\sqrt{2}\sqrt{-\frac{i}{-3 i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 367: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 64 leaves, 1 step):

$$\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 142 leaves):

$$-\left(\left(i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2}{-3-i\sqrt{7}}}\,x\right],\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right]\right)\Big/\left(\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)\right)$$

Problem 368: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 168 leaves, 3 steps):

$$\frac{1}{4}\sqrt{\frac{5}{77}}\ \text{ArcTan}\Big[\frac{2\sqrt{\frac{11}{35}}\ x}{\sqrt{4+3x^2+x^4}}\Big]-\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\ \text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{x}{\sqrt{2}}\Big],\frac{1}{8}\Big]}{6\sqrt{2}\ \sqrt{4+3x^2+x^4}}+$$
$$\frac{17\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\ \text{EllipticPi}\Big[-\frac{9}{280},2\,\text{ArcTan}\Big[\frac{x}{\sqrt{2}}\Big],\frac{1}{8}\Big]}{84\sqrt{2}\ \sqrt{4+3x^2+x^4}}$$

Result (type 4, 159 leaves):

$$-\left(\left(\mathrm{i}\sqrt{1-\frac{2x^2}{-3-\mathrm{i}\sqrt{7}}}\ \sqrt{1-\frac{2x^2}{-3+\mathrm{i}\sqrt{7}}}\ \text{EllipticPi}\Big[-\frac{5}{14}\left(-3-\mathrm{i}\sqrt{7}\right),\right.\right.$$
$$\left.\left.\mathrm{i}\,\text{ArcSinh}\Big[\sqrt{-\frac{2}{-3-\mathrm{i}\sqrt{7}}}\ x\Big],\frac{-3-\mathrm{i}\sqrt{7}}{-3+\mathrm{i}\sqrt{7}}\Big]\right)\Big/\left(7\sqrt{2}\ \sqrt{-\frac{1}{-3-\mathrm{i}\sqrt{7}}}\ \sqrt{4+3x^2+x^4}\right)\right)$$

Problem 369: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^2\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 286 leaves, 6 steps):

$$-\frac{5x\sqrt{4+3x^2+x^4}}{616\left(2+x^2\right)}+\frac{25x\sqrt{4+3x^2+x^4}}{616\left(7+5x^2\right)}+\frac{37\sqrt{\frac{5}{77}}\ \text{ArcTan}\Big[\frac{2\sqrt{\frac{11}{35}}\ x}{\sqrt{4+3x^2+x^4}}\Big]}{2464}+$$
$$\frac{5\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\ \text{EllipticE}\Big[2\,\text{ArcTan}\Big[\frac{x}{\sqrt{2}}\Big],\frac{1}{8}\Big]}{308\sqrt{2}\ \sqrt{4+3x^2+x^4}}-$$
$$\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\ \text{EllipticF}\Big[2\,\text{ArcTan}\Big[\frac{x}{\sqrt{2}}\Big],\frac{1}{8}\Big]}{42\sqrt{2}\ \sqrt{4+3x^2+x^4}}+$$
$$\frac{629\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\ \text{EllipticPi}\Big[-\frac{9}{280},2\,\text{ArcTan}\Big[\frac{x}{\sqrt{2}}\Big],\frac{1}{8}\Big]}{51744\sqrt{2}\ \sqrt{4+3x^2+x^4}}$$

Result (type 4, 481 leaves):

$$\frac{1}{17248\sqrt{-\frac{i}{-3i+\sqrt{7}}}\left(7+5x^2\right)\sqrt{4+3x^2+x^4}}$$

$$\left(700\sqrt{-\frac{i}{-3i+\sqrt{7}}}\;x\left(4+3x^2+x^4\right)+35\left(3i+\sqrt{7}\right)\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\right.$$
$$\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\;x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-\right.$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\;x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)+98i\left(7+5x^2\right)$$
$$\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\;x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-$$
$$74i\left(7+5x^2\right)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}$$
$$\left.\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\;x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 370: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}}\,dx$$

Optimal (type 4, 314 leaves, 7 steps):

$$-\frac{555x\sqrt{4+3x^2+x^4}}{758912\left(2+x^2\right)}+\frac{25x\sqrt{4+3x^2+x^4}}{1232\left(7+5x^2\right)^2}+\frac{2775x\sqrt{4+3x^2+x^4}}{758912\left(7+5x^2\right)}-$$
$$\frac{3285\sqrt{\frac{5}{77}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]}{3035648}+\frac{555\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{379456\sqrt{2}\sqrt{4+3x^2+x^4}}-$$
$$\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{8624\sqrt{2}\sqrt{4+3x^2+x^4}}-$$
$$\frac{18615\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\;\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{21249536\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 308 leaves):

$$\left(\frac{700\,x\,(1393+555\,x^2)\,(4+3\,x^2+x^4)}{(7+5\,x^2)^2}+i\,\sqrt{6+2\,i\,\sqrt{7}}\,\sqrt{1-\frac{2\,i\,x^2}{-3\,i+\sqrt{7}}}\,\sqrt{1+\frac{2\,i\,x^2}{3\,i+\sqrt{7}}}\left(3885\,(3-i\,\sqrt{7})\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+(-9401+3885\,i\,\sqrt{7})\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]+6570\,\text{EllipticPi}\left[\frac{5}{14}\,(3+i\,\sqrt{7}),\,i\,\text{ArcSinh}\left[\sqrt{-\frac{2\,i}{-3\,i+\sqrt{7}}}\,x\right],\frac{3\,i-\sqrt{7}}{3\,i+\sqrt{7}}\right]\right)\right)\Big/\left(21\,249\,536\,\sqrt{4+3\,x^2+x^4}\right)$$

Problem 371: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(7+5\,x^2)^5}{(4+3\,x^2+x^4)^{3/2}}\,dx$$

Optimal (type 4, 219 leaves, 6 steps):

$$\frac{x\,(99\,493+45\,779\,x^2)}{28\,\sqrt{4+3\,x^2+x^4}}+\frac{5000}{3}\,x\,\sqrt{4+3\,x^2+x^4}+625\,x^3\,\sqrt{4+3\,x^2+x^4}-\frac{220\,779\,x\,\sqrt{4+3\,x^2+x^4}}{28\,(2+x^2)}+\frac{220\,779\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14\,\sqrt{2}\,\sqrt{4+3\,x^2+x^4}}-\frac{130\,729\,(2+x^2)\,\sqrt{\frac{4+3\,x^2+x^4}{(2+x^2)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{12\,\sqrt{2}\,\sqrt{4+3\,x^2+x^4}}$$

Result (type 4, 339 leaves):

$$\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(858479+767337x^2+297500x^4+52500x^6\right)+\right.$$
$$662337\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}$$
$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-$$
$$\sqrt{2}\left(975947i+662337\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Bigg/\left(336\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 372: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^4}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 200 leaves, 5 steps):

$$\frac{x\left(2719-4023x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{625}{3}x\sqrt{4+3x^2+x^4}+\frac{14523x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}-$$
$$\frac{14523\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14\sqrt{2}\sqrt{4+3x^2+x^4}}+$$
$$\frac{4243\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{12\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 333 leaves):

$$\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(78157+40431x^2+17500x^4\right)-43569\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(186179i+43569\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Bigg/\left(336\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 373: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(7+5x^2\right)^3}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 181 leaves, 4 steps):

$$-\frac{x\left(2323+949x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{4449x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}-\frac{4449\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{973\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{4\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 328 leaves):

$$\left(-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(2323+949x^2\right)-4449\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(3899i+4449\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Bigg/\left(112\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 374: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(7+5 x^2\right)^2}{\left(4+3 x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 181 leaves, 4 steps):

$$-\frac{x\left(9-113 x^2\right)}{28 \sqrt{4+3 x^2+x^4}}-\frac{113 x \sqrt{4+3 x^2+x^4}}{28\left(2+x^2\right)}+\frac{113\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{14 \sqrt{2} \sqrt{4+3 x^2+x^4}}+\frac{9\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{4 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 329 leaves):

$$\left(4 \sqrt{-\frac{i}{-3 i+\sqrt{7}}}\, x\left(-9+113 x^2\right)+113 \sqrt{2}\left(3 i+\sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]-\sqrt{2}\left(1043 i+113 \sqrt{7}\right) \sqrt{\frac{-3 i+\sqrt{7}-2 i x^2}{-3 i+\sqrt{7}}} \sqrt{\frac{3 i+\sqrt{7}+2 i x^2}{3 i+\sqrt{7}}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}}\, x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right) \Bigg/ \left(112 \sqrt{-\frac{i}{-3 i+\sqrt{7}}} \sqrt{4+3 x^2+x^4}\right)$$

Problem 375: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{7+5 x^2}{\left(4+3 x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 181 leaves, 4 steps):

$$\frac{x\left(53+19 x^2\right)}{28 \sqrt{4+3 x^2+x^4}}-\frac{19 x \sqrt{4+3 x^2+x^4}}{28\left(2+x^2\right)}+\frac{19\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\, \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{14 \sqrt{2} \sqrt{4+3 x^2+x^4}}-\frac{3\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}}\, \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{4 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 329 leaves):

$$\left(4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(53+19x^2\right)+19\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-\sqrt{2}\left(49i+19\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(112\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 376: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 181 leaves, 4 steps):

$$-\frac{x\left(1+3x^2\right)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}-\frac{3\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{14\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{4\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 328 leaves):

$$\left(-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(1+3x^2\right)-3\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(-7i+3\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)\Big/\left(112\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}\right)$$

Problem 377: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5x^2\right)\left(4+3x^2+x^4\right)^{3/2}}\,dx$$

Optimal (type 4, 284 leaves, 8 steps):

$$-\frac{x\left(13+4x^2\right)}{308\sqrt{4+3x^2+x^4}}+\frac{x\sqrt{4+3x^2+x^4}}{77\left(2+x^2\right)}+\frac{25}{176}\sqrt{\frac{5}{77}}\,\text{ArcTan}\left[\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right]-\frac{\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{77\sqrt{4+3x^2+x^4}}-\frac{\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{12\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{425\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{9}{280},2\,\text{ArcTan}\left[\frac{x}{\sqrt{2}}\right],\frac{1}{8}\right]}{3696\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Result (type 4, 483 leaves):

$$\frac{1}{616\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}\left(-26\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x-8\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x^3-2\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]+\sqrt{2}\left(7i+2\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]-25i\sqrt{2}\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\text{EllipticPi}\left[\frac{5}{14}\left(3+i\sqrt{7}\right),i\,\text{ArcSinh}\left[\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right],\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right]\right)$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5 x^2\right)^2 \left(4+3 x^2+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 312 leaves, 15 steps):

$$\frac{x \left(24+37 x^2\right)}{13552 \sqrt{4+3 x^2+x^4}}-\frac{199 x \sqrt{4+3 x^2+x^4}}{27104 \left(2+x^2\right)}+\frac{625 x \sqrt{4+3 x^2+x^4}}{27104 \left(7+5 x^2\right)}+$$
$$\frac{575 \sqrt{\frac{5}{77}} \text{ArcTan}\left[\frac{2 \sqrt{\frac{11}{35}} x}{\sqrt{4+3 x^2+x^4}}\right]}{108416}+\frac{199 \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{13552 \sqrt{2} \sqrt{4+3 x^2+x^4}}-$$
$$\frac{2 \sqrt{2} \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{231 \sqrt{4+3 x^2+x^4}}+$$
$$\frac{9775 \left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticPi}\left[-\frac{9}{280}, 2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{2276736 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 311 leaves):

$$\frac{1}{758912 \left(7+5 x^2\right) \sqrt{4+3 x^2+x^4}}$$
$$\left(28 x \left(2836+2633 x^2+995 x^4\right)+i \sqrt{6+2 i \sqrt{7}} \left(7+5 x^2\right) \sqrt{1-\frac{2 i x^2}{-3 i+\sqrt{7}}} \sqrt{1+\frac{2 i x^2}{3 i+\sqrt{7}}}\right.$$
$$\left(1393 \left(3-i \sqrt{7}\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+\right.$$
$$7 \left(101+199 i \sqrt{7}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]-$$
$$\left.\left.1150 \text{EllipticPi}\left[\frac{5}{14} \left(3+i \sqrt{7}\right), i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right)\right)$$

Problem 379: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(7+5 x^2\right)^3 \left(4+3 x^2+x^4\right)^{3/2}} \, dx$$

Optimal (type 4, 340 leaves, 22 steps):

$$\frac{x\left(548+139 x^2\right)}{596288 \sqrt{4+3 x^2+x^4}}-\frac{18159 x \sqrt{4+3 x^2+x^4}}{33392128\left(2+x^2\right)}+\frac{625 x \sqrt{4+3 x^2+x^4}}{54208\left(7+5 x^2\right)^2}+\frac{51875 x \sqrt{4+3 x^2+x^4}}{33392128\left(7+5 x^2\right)}-$$

$$\frac{529425 \sqrt{\frac{5}{77}} \text{ArcTan}\left[\frac{2 \sqrt{\frac{11}{35}} x}{\sqrt{4+3 x^2+x^4}}\right]}{133568512}+\frac{18159\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{16696064 \sqrt{2} \sqrt{4+3 x^2+x^4}}+$$

$$\frac{843\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{379456 \sqrt{2} \sqrt{4+3 x^2+x^4}}-$$

$$\frac{3000075\left(2+x^2\right) \sqrt{\frac{4+3 x^2+x^4}{\left(2+x^2\right)^2}} \text{EllipticPi}\left[-\frac{9}{280}, 2 \text{ArcTan}\left[\frac{x}{\sqrt{2}}\right], \frac{1}{8}\right]}{934979584 \sqrt{2} \sqrt{4+3 x^2+x^4}}$$

Result (type 4, 320 leaves):

$$\frac{1}{934979584\left(7+5 x^2\right)^2 \sqrt{4+3 x^2+x^4}}\left(28 x\left(4496212+5811451 x^2+2838330 x^4+453975 x^6\right)+\right.$$

$$3 i \sqrt{6+2 i \sqrt{7}}\left(7+5 x^2\right)^2 \sqrt{1-\frac{2 i x^2}{-3 i+\sqrt{7}}} \sqrt{1+\frac{2 i x^2}{3 i+\sqrt{7}}}$$

$$\left(42371\left(3-i \sqrt{7}\right) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+\right.$$

$$7 i\left(23633 i+6053 \sqrt{7}\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]+$$

$$\left.\left.352950 \text{EllipticPi}\left[\frac{5}{14}\left(3+i \sqrt{7}\right), i \text{ArcSinh}\left[\sqrt{-\frac{2 i}{-3 i+\sqrt{7}}} x\right], \frac{3 i-\sqrt{7}}{3 i+\sqrt{7}}\right]\right)\right)$$

Problem 380: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\left(d+e x^2\right)^3}{\sqrt{a+b x^2+c x^4}} \, dx$$

Optimal (type 4, 467 leaves, 5 steps):

$$\frac{e^2\left(15\,c\,d-4\,b\,e\right)x\sqrt{a+b\,x^2+c\,x^4}}{15\,c^2}+\frac{e^3\,x^3\sqrt{a+b\,x^2+c\,x^4}}{5\,c}+$$

$$\frac{e\left(45\,c^2\,d^2+8\,b^2\,e^2-3\,c\,e\left(10\,b\,d+3\,a\,e\right)\right)x\sqrt{a+b\,x^2+c\,x^4}}{15\,c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$

$$\left(a^{1/4}\,e\left(45\,c^2\,d^2+8\,b^2\,e^2-3\,c\,e\left(10\,b\,d+3\,a\,e\right)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(15\,c^{11/4}\sqrt{a+b\,x^2+c\,x^4}\right)+$$

$$\left(a^{1/4}\left(\frac{\sqrt{c}\left(15\,c^2\,d^3-15\,a\,c\,d\,e^2+4\,a\,b\,e^3\right)}{\sqrt{a}}+e\left(45\,c^2\,d^2+8\,b^2\,e^2-3\,c\,e\left(10\,b\,d+3\,a\,e\right)\right)\right)\right.$$

$$\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(30\,c^{11/4}\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 1825 leaves):

$$\left(-\frac{e^2\left(-15\,c\,d+4\,b\,e\right)x}{15\,c^2}+\frac{e^3\,x^3}{5\,c}\right)\sqrt{a+b\,x^2+c\,x^4}+$$

$$\frac{1}{15\,c^2}\left(\left(\left(45\,i\,c\left(-b+\sqrt{b^2-4\,a\,c}\right)d^2\,e\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/$$

$$\left(2\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)-$$

$$\left(15\,i\,b\left(-b+\sqrt{b^2-4\,a\,c}\right)d\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$

$$\left(\text{EllipticE}\left[\text{i} \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] - \text{EllipticF}\left[\text{i} \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\Bigg) \Bigg/ \left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\Bigg(9 \,\text{i}\, a \left(-b+\sqrt{b^2-4 a c}\right) e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}} \left(\text{EllipticE}\left[\text{i} \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] - \text{EllipticF}\left[\text{i} \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\Bigg) \Bigg/ \left(2 \sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) +$$

$$\Bigg(2 \,\text{i} \sqrt{2}\, b^2 \left(-b+\sqrt{b^2-4 a c}\right) e^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}} \left(\text{EllipticE}\left[\text{i} \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right] - \text{EllipticF}\left[\text{i} \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\Bigg) \Bigg/ \left(c \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) -$$

$$\Bigg(15 \,\text{i}\, c^2 d^3 \sqrt{1-\frac{2 c x^2}{-b-\sqrt{b^2-4 a c}}} \sqrt{1-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\; \text{EllipticF}\left[\text{i} \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}}\, x\right], \frac{-b-\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\Bigg) \Bigg/ \left(\sqrt{2} \sqrt{-\frac{c}{-b-\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}\right) +$$

$$\left(15\,i\,a\,c\,d\,e^2\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)-$$
$$\left(2\,i\sqrt{2}\,a\,b\,e^3\sqrt{1-\frac{2\,c\,x^2}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{1-\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\right.$$
$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\,x\right],\frac{-b-\sqrt{b^2-4\,a\,c}}{-b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left.\left(\sqrt{-\frac{c}{-b-\sqrt{b^2-4\,a\,c}}}\sqrt{a+b\,x^2+c\,x^4}\right)\right)$$

Problem 381: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e\,x^2\right)^2}{\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 356 leaves, 4 steps):

$$\frac{e^2\,x\sqrt{a+b\,x^2+c\,x^4}}{3\,c}+\frac{2\,e\,(3\,c\,d-b\,e)\,x\sqrt{a+b\,x^2+c\,x^4}}{3\,c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-$$
$$\left(2\,a^{1/4}\,e\,(3\,c\,d-b\,e)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$
$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(3\,c^{7/4}\sqrt{a+b\,x^2+c\,x^4}\right)+$$
$$\left(a^{1/4}\left(2\,e\,(3\,c\,d-b\,e)+\frac{\sqrt{c}\left(3\,c\,d^2-a\,e^2\right)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$
$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(6\,c^{7/4}\sqrt{a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 488 leaves):

$$\frac{1}{6 c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \sqrt{a+b x^2+c x^4}} \left(2 c \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, e^2 x \left(a+b x^2+c x^4\right) - i \left(-b+\sqrt{b^2-4 a c}\right) e \left(-3 c d+b e\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right] + i \left(-3 c^2 d^2+b \left(-b+\sqrt{b^2-4 a c}\right) e^2+c e \left(3 b d-3 \sqrt{b^2-4 a c}\, d+a e\right)\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

Problem 382: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x^2}{\sqrt{a+b x^2+c x^4}}\, dx$$

Optimal (type 4, 283 leaves, 3 steps):

$$\frac{e x \sqrt{a+b x^2+c x^4}}{\sqrt{c} \left(\sqrt{a}+\sqrt{c}\, x^2\right)} - \left(a^{1/4} e \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(c^{3/4} \sqrt{a+b x^2+c x^4}\right) + \left(a^{1/4} \left(\frac{\sqrt{c}\, d}{\sqrt{a}}+e\right) \left(\sqrt{a}+\sqrt{c}\, x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(2 c^{3/4} \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 302 leaves):

$$\left(i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right.$$
$$\left(\left(-b+\sqrt{b^2-4ac}\right)e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+\right.$$
$$\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)$$
$$\left.\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)\right)\Big/$$
$$\left(2\sqrt{2}\,c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}\right)$$

Problem 383: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 401 leaves, 3 steps):

$$\frac{\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right]}{2\sqrt{d}\,\sqrt{cd^2-bde+ae^2}}+$$
$$\left(c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$
$$\left(2a^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+bx^2+cx^4}\right)-$$
$$\left(a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de},\right.\right.$$
$$\left.\left.2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(4c^{1/4}d\left(cd^2-ae^2\right)\sqrt{a+bx^2+cx^4}\right)$$

Result (type 4, 214 leaves):

$$-\left(\left(\left(i \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\right.\right.\right.$$

$$\left.\left.\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4 a c}\right) e}{2 c d}, i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right) \Big/$$

$$\left.\left(\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} d \sqrt{a+b x^2+c x^4}\right)\right)$$

Problem 384: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e x^2\right)^2 \sqrt{a+b x^2+c x^4}} \, dx$$

Optimal (type 4, 718 leaves, 6 steps):

$$-\frac{\sqrt{c} e x \sqrt{a+b x^2+c x^4}}{2 d \left(c d^2-b d e+a e^2\right) \left(\sqrt{a}+\sqrt{c} x^2\right)}+\frac{e^2 x \sqrt{a+b x^2+c x^4}}{2 d \left(c d^2-b d e+a e^2\right) \left(d+e x^2\right)}+$$

$$\frac{\sqrt{e} \left(3 c d^2-e (2 b d-a e)\right) \text{ArcTan}\left[\frac{\sqrt{c d^2-b d e+a e^2} x}{\sqrt{d} \sqrt{e} \sqrt{a+b x^2+c x^4}}\right]}{4 d^{3/2} \left(c d^2-b d e+a e^2\right)^{3/2}}+$$

$$\left(a^{1/4} c^{1/4} e \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$

$$\left(2 d \left(c d^2-b d e+a e^2\right) \sqrt{a+b x^2+c x^4}\right)+$$

$$\left(c^{1/4} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$

$$\left(2 a^{1/4} d \left(\sqrt{c} d-\sqrt{a} e\right) \sqrt{a+b x^2+c x^4}\right)-$$

$$\left(\left(\sqrt{c} d+\sqrt{a} e\right) \left(3 c d^2-e (2 b d-a e)\right) \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{c} d-\sqrt{a} e\right)^2}{4 \sqrt{a} \sqrt{c} d e}, 2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/$$

$$\left(8 a^{1/4} c^{1/4} d^2 \left(\sqrt{c} d-\sqrt{a} e\right) \left(c d^2-b d e+a e^2\right) \sqrt{a+b x^2+c x^4}\right)$$

Result (type 4, 1069 leaves):

$$\frac{1}{8\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,d\left(c d^3+d e\left(-b d+a e\right)\right)\left(d+e x^2\right)\sqrt{a+b x^2+c x^4}}$$

$$\left(4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,d e^2 x\left(a+b x^2+c x^4\right)+\right.$$

$$i\sqrt{2}\left(b-\sqrt{b^2-4ac}\right) d e\sqrt{\frac{b+\sqrt{b^2-4ac}+2 c x^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4ac}}}$$

$$\left(d+e x^2\right)\left(\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-\right.$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)+$$

$$2 i\sqrt{2}\,c d^2\sqrt{\frac{b+\sqrt{b^2-4ac}+2 c x^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4ac}}}\left(d+e x^2\right)$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-$$

$$6 i\sqrt{2}\,c d^2\sqrt{\frac{b+\sqrt{b^2-4ac}+2 c x^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4ac}}}\left(d+e x^2\right)$$

$$\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2 c d},i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+$$

$$4 i\sqrt{2}\,b d e\sqrt{\frac{b+\sqrt{b^2-4ac}+2 c x^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4ac}}}\left(d+e x^2\right)$$

$$\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2 c d},i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]-$$

$$2 i\sqrt{2}\,a e^2\sqrt{\frac{b+\sqrt{b^2-4ac}+2 c x^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4ac}}}\left(d+e x^2\right)$$

$$\left.\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2 c d},i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)$$

Problem 385: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^3}{\sqrt{a+b x^2-c x^4}} \, dx$$

Optimal (type 4, 553 leaves, 6 steps):

$$-\frac{e^2 (15 c d+4 b e) x \sqrt{a+b x^2-c x^4}}{15 c^2}-\frac{e^3 x^3 \sqrt{a+b x^2-c x^4}}{5 c}-\left(\left(b-\sqrt{b^2+4 a c}\right) \sqrt{b+\sqrt{b^2+4 a c}}\; e \left(45 c^2 d^2+8 b^2 e^2+3 c e (10 b d+3 a e)\right) \sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\; x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right) \Big/ \left(30 \sqrt{2}\; c^{7/2} \sqrt{a+b x^2-c x^4}\right)+\left(\left(b-\sqrt{b^2+4 a c}\right) \sqrt{b+\sqrt{b^2+4 a c}} \left(\frac{2 c \left(15 c^2 d^3+15 a c d e^2+4 a b e^3\right)}{b-\sqrt{b^2+4 a c}}+e \left(45 c^2 d^2+8 b^2 e^2+3 c e (10 b d+3 a e)\right)\right) \sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\; x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right) \Big/ \left(30 \sqrt{2}\; c^{7/2} \sqrt{a+b x^2-c x^4}\right)$$

Result (type 4, 596 leaves):

$$\frac{1}{60\, c^3 \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\ \sqrt{a+b x^2-c x^4}}$$
$$\left(-4 c \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\ e^2 x \left(a+b x^2-c x^4\right) \left(4 b e+3 c \left(5 d+e x^2\right)\right)-\right.$$
$$i \sqrt{2} \left(-b+\sqrt{b^2+4 a c}\right) e \left(45 c^2 d^2+8 b^2 e^2+3 c e \left(10 b d+3 a e\right)\right) \sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}}$$
$$\sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\ \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\ x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+$$
$$i \sqrt{2} \left(-30 c^3 d^3+8 b^2 \left(-b+\sqrt{b^2+4 a c}\right) e^3+15 c^2 d e \left(-3 b d+3 \sqrt{b^2+4 a c}\ d-2 a e\right)+\right.$$
$$\left.c e^2 \left(-30 b^2 d+30 b \sqrt{b^2+4 a c}\ d-17 a b e+9 a \sqrt{b^2+4 a c}\ e\right)\right) \sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}}$$
$$\left.\sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\ \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\ x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)$$

Problem 386: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(d+e x^2\right)^2}{\sqrt{a+b x^2-c x^4}}\, dx$$

Optimal (type 4, 454 leaves, 5 steps):

$$-\frac{e^2 x \sqrt{a+b x^2-c x^4}}{3 c}-\left(\left(b-\sqrt{b^2+4 a c}\right) \sqrt{b+\sqrt{b^2+4 a c}}\; e\, (3 c d+b e) \sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\; \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right) \Big/ \left(3 \sqrt{2}\, c^{5/2} \sqrt{a+b x^2-c x^4}\right)+\left(\sqrt{b+\sqrt{b^2+4 a c}} \left(3 c^2 d^2+b \left(b-\sqrt{b^2+4 a c}\right) e^2+c e \left(3 b d-3 \sqrt{b^2+4 a c}\, d+a e\right)\right) \sqrt{1-\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\; \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} \sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right) \Big/ \left(3 \sqrt{2}\, c^{5/2} \sqrt{a+b x^2-c x^4}\right)$$

Result (type 4, 503 leaves):

$$\frac{1}{6 c^2 \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}} \left(2 c \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\; e^2 x \left(-a-b x^2+c x^4\right)- i \sqrt{2} \left(-b+\sqrt{b^2+4 a c}\right) e\, (3 c d+b e) \sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\; \text{EllipticE}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\; x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]+i \sqrt{2} \left(-3 c^2 d^2+b \left(-b+\sqrt{b^2+4 a c}\right) e^2-c e \left(3 b d-3 \sqrt{b^2+4 a c}\, d+a e\right)\right) \sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}}\; \text{EllipticF}\left[i\, \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\; x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]\right)$$

Problem 387: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{d+e x^2}{\sqrt{a+b x^2-c x^4}}\, dx$$

Optimal (type 4, 385 leaves, 4 steps):

$$-\left(\left(\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\; e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\Big/\left(2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}\right)\right)+$$

$$\left(\sqrt{b+\sqrt{b^2+4ac}}\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\Big/\left(2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}\right)$$

Result (type 4, 293 leaves) :

$$-\left(\left(i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(\left(-b+\sqrt{b^2+4ac}\right)e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\;x\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]+\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\;x\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\right)\Big/\left(2\sqrt{2}\,c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}\right)\right)$$

Problem 388: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+ex^2\right)\sqrt{a+bx^2-cx^4}}\,dx$$

Optimal (type 4, 197 leaves, 2 steps) :

$$\left(\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\;\text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\Big/\left(\sqrt{2}\sqrt{c}\,d\sqrt{a+bx^2-cx^4}\right)$$

Result (type 4, 205 leaves) :

$$-\left(\left(\left(i\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4\,a\,c}\right)e}{2\,c\,d},\,i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,x\right],\,-\frac{b+\sqrt{b^2+4\,a\,c}}{-b+\sqrt{b^2+4\,a\,c}}\right]\right)\Big/\left(\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\,d\,\sqrt{a+b\,x^2-c\,x^4}\right)\right)\right)$$

Problem 389: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+e\,x^2\right)^2\sqrt{a+b\,x^2-c\,x^4}}\,dx$$

Optimal (type 4, 718 leaves, 8 steps):

$$-\frac{e^2\,x\,\sqrt{a+b\,x^2-c\,x^4}}{2\,d\,\left(c\,d^2+b\,d\,e-a\,e^2\right)\left(d+e\,x^2\right)}+$$
$$\left(\left(b-\sqrt{b^2+4\,a\,c}\right)\sqrt{b+\sqrt{b^2+4\,a\,c}}\;e\,\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\,\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)\Big/$$
$$\left(4\,\sqrt{2}\,\sqrt{c}\,d\,\left(c\,d^2+e\,\left(b\,d-a\,e\right)\right)\sqrt{a+b\,x^2-c\,x^4}\right)-$$
$$\left(\sqrt{b+\sqrt{b^2+4\,a\,c}}\,\left(2\,c\,d+\left(b-\sqrt{b^2+4\,a\,c}\right)e\right)\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\,\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)\Big/$$
$$\left(4\,\sqrt{2}\,\sqrt{c}\,d\,\left(c\,d^2+e\,\left(b\,d-a\,e\right)\right)\sqrt{a+b\,x^2-c\,x^4}\right)+$$
$$\left(\sqrt{b+\sqrt{b^2+4\,a\,c}}\,\left(3\,c\,d^2+e\,\left(2\,b\,d-a\,e\right)\right)\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\,\text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4\,a\,c}\right)e}{2\,c\,d},\,\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\,\frac{b+\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)\Big/$$
$$\left(2\,\sqrt{2}\,\sqrt{c}\,d^2\,\left(c\,d^2+e\,\left(b\,d-a\,e\right)\right)\sqrt{a+b\,x^2-c\,x^4}\right)$$

Result (type 4, 1341 leaves):

$$-\frac{e^2 x \sqrt{a+b x^2-c x^4}}{2 d \left(c d^2+b d e-a e^2\right) \left(d+e x^2\right)}+\frac{1}{2 c d^2+2 b d e-a e^2-2 c d e x^2-c e^2 x^4}$$

$$\left(d+e x^2\right) \sqrt{a+b x^2-c x^4} \left(-\frac{c}{2 \left(c d^2+b d e-a e^2\right) \sqrt{a+b x^2-c x^4}}-\right.$$

$$\left.\frac{c e x^2}{2 d \left(c d^2+b d e-a e^2\right) \sqrt{a+b x^2-c x^4}}+\frac{3 c d^2+2 b d e-a e^2}{2 d \left(c d^2+b d e-a e^2\right) \left(d+e x^2\right) \sqrt{a+b x^2-c x^4}}\right)$$

$$\left(\left(\left(i \left(-b+\sqrt{b^2+4 a c}\right) e \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\right.\right.\right.$$

$$\left(\text{EllipticE}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]-\right.$$

$$\left.\left.\text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]\right)\right)\Big/$$

$$\left(2 \sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}\right)+\left(i c d \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}}\right.$$

$$\sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\ \text{EllipticF}\left[i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],\right.$$

$$\left.\left.-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]\right)\Big/\left(\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}\right)-$$

$$\left(3 i c d \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\ \text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4 a c}\right) e}{2 c d},\right.\right.$$

$$\left.\left.i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]\right)\Big/$$

$$\left(\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} \sqrt{a+b x^2-c x^4}\right)-$$

$$\left(i \sqrt{2}\, b e \sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2+4 a c}}} \sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\ \text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(b+\sqrt{b^2+4 a c}\right) e}{2 c d}, i \,\text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}\, x\right],-\frac{b+\sqrt{b^2+4 a c}}{-b+\sqrt{b^2+4 a c}}\right]\right)\Big/$$

$$\left(\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\ \sqrt{a+b\,x^2-c\,x^4}\right)+\left(\mathrm{i}\,a\,e^2\,\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2+4\,a\,c}}}\ \sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}\right.$$

$$\text{EllipticPi}\left[-\frac{\left(b+\sqrt{b^2+4\,a\,c}\right)\,e}{2\,c\,d},\ \mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\ x\right],\right.$$

$$\left.\left.-\frac{b+\sqrt{b^2+4\,a\,c}}{-b+\sqrt{b^2+4\,a\,c}}\right]\right)\Big/\left(\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2+4\,a\,c}}}\ d\,\sqrt{a+b\,x^2-c\,x^4}\right)\Bigg)$$

Problem 390: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e\,x^2}{\sqrt{-a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 479 leaves, 5 steps):

$$\frac{\left(b-\sqrt{b^2+4\,a\,c}\right)\,e\,x\,\left(1+\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}\right)}{2\,c\,\sqrt{-a+b\,x^2+c\,x^4}}-$$

$$\left(\left(b-\sqrt{b^2+4\,a\,c}\right)\,\sqrt{b+\sqrt{b^2+4\,a\,c}}\ e\,\left(1+\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}\right)\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\right.\right.$$

$$\left.\left.-\frac{2\,\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)\Big/\left(2\,\sqrt{2}\,c^{3/2}\,\sqrt{\frac{1+\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}{1+\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}}\ \sqrt{-a+b\,x^2+c\,x^4}\right)+$$

$$\left(\sqrt{b+\sqrt{b^2+4\,a\,c}}\ d\,\left(1+\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}\right)\,\text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4\,a\,c}}}\right],\right.\right.$$

$$\left.\left.-\frac{2\,\sqrt{b^2+4\,a\,c}}{b-\sqrt{b^2+4\,a\,c}}\right]\right)\Big/\left(\sqrt{2}\,\sqrt{c}\,\sqrt{\frac{1+\frac{2\,c\,x^2}{b-\sqrt{b^2+4\,a\,c}}}{1+\frac{2\,c\,x^2}{b+\sqrt{b^2+4\,a\,c}}}}\ \sqrt{-a+b\,x^2+c\,x^4}\right)$$

Result (type 4, 304 leaves):

$$\left(\mathrm{i}\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\right.$$
$$\left(\left(-b+\sqrt{b^2+4ac}\right)e\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]+\right.$$
$$\left(-2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)$$
$$\left.\left.\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\right)\Big/$$
$$\left(2\sqrt{2}\,c\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{-a+bx^2+cx^4}\right)$$

Problem 391: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\left(d+ex^2\right)\sqrt{-a+bx^2+cx^4}}\,dx$$

Optimal (type 4, 204 leaves, 2 steps):

$$\left(\sqrt{-b+\sqrt{b^2+4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\text{EllipticPi}\left[\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd},\right.\right.$$
$$\left.\left.\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{-b+\sqrt{b^2+4ac}}}\right],\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right]\right)\Big/\left(\sqrt{2}\sqrt{c}\,d\sqrt{-a+bx^2+cx^4}\right)$$

Result (type 4, 216 leaves):

$$-\left(\left(\mathrm{i}\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\right.\right.$$
$$\left.\text{EllipticPi}\left[\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd},\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right],\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right]\right)\Big/$$
$$\left.\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,d\sqrt{-a+bx^2+cx^4}\right)\right)$$

Problem 392: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$$

Optimal (type 4, 293 leaves, 3 steps):

$$-\frac{e\,x\,\sqrt{-a+b\,x^2-c\,x^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\left(a^{1/4}\,e\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a-b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2+\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(c^{3/4}\,\sqrt{-a+b\,x^2-c\,x^4}\right)+\left(a^{1/4}\,\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a-b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2+\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(2\,c^{3/4}\,\sqrt{-a+b\,x^2-c\,x^4}\right)$$

Result (type 4, 295 leaves):

$$-\left(\left(\mathrm{i}\,\sqrt{1+\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\left(\left(-b+\sqrt{b^2-4\,a\,c}\right)\,e\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+\left(2\,c\,d+\left(b-\sqrt{b^2-4\,a\,c}\right)\,e\right)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(2\,\sqrt{2}\,c\,\sqrt{-\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{-a+b\,x^2-c\,x^4}\right)\right)$$

Problem 393: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(d+e\,x^2\right)\,\sqrt{-a+b\,x^2-c\,x^4}}\,dx$$

Optimal (type 4, 412 leaves, 3 steps):

$$\frac{\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{-c d^2-e (b d+a e)}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a+b x^2-c x^4}}\right]}{2\sqrt{d}\,\sqrt{-c d^2-e\,(b d+a e)}}+$$

$$\left(c^{1/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/$$

$$\left(2\,a^{1/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{-a+b x^2-c x^4}\right)-$$

$$\left(a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,d e},\right.\right.$$

$$\left.\left.2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(4\,c^{1/4}\,d\left(c d^2-a e^2\right)\sqrt{-a+b x^2-c x^4}\right)$$

Result (type 4, 207 leaves):

$$-\left(\left(\left(i\sqrt{1+\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}}\,\sqrt{1-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\,\text{EllipticPi}\left[\right.\right.\right.\right.$$

$$\left.\left.-\frac{\left(b+\sqrt{b^2-4 a c}\right)e}{2 c d},\,i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2-4 a c}}}\,x\right],-\frac{b+\sqrt{b^2-4 a c}}{-b+\sqrt{b^2-4 a c}}\right]\right)\Big/$$

$$\left.\left(\sqrt{2}\,\sqrt{-\frac{c}{b+\sqrt{b^2-4 a c}}}\,d\,\sqrt{-a+b x^2-c x^4}\right)\right)$$

Problem 394: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e x^2\right)^3}{\sqrt{2+3 x^2+x^4}}\,dx$$

Optimal (type 4, 229 leaves, 5 steps):

$$\frac{3 e\left(5 d^2-10 d e+6 e^2\right)x\left(2+x^2\right)}{5\sqrt{2+3 x^2+x^4}}+\frac{1}{5}\left(5 d-4 e\right)e^2 x\sqrt{2+3 x^2+x^4}+\frac{1}{5}e^3 x^3\sqrt{2+3 x^2+x^4}-$$

$$\frac{1}{5\sqrt{2+3 x^2+x^4}}3\sqrt{2}\,e\left(5 d^2-10 d e+6 e^2\right)\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]+$$

$$\frac{\left(5 d^3-10 d e^2+8 e^3\right)\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{5\sqrt{2}\,\sqrt{2+3 x^2+x^4}}$$

Result (type 4, 154 leaves):

$$\frac{1}{5\sqrt{2+3x^2+x^4}}\left(e^2 x\left(2+3x^2+x^4\right)\left(5d+e\left(-4+x^2\right)\right)-3\,i\,e\left(5d^2-10de+6e^2\right)\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-5\,i\left(d^3-3d^2e+4de^2-2e^3\right)\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 395: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\left(d+e x^2\right)^2}{\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 168 leaves, 4 steps):

$$\frac{2(d-e)\,e\,x\left(2+x^2\right)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}e^2 x\sqrt{2+3x^2+x^4}-\frac{2\sqrt{2}\,(d-e)\,e\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{\left(3d^2-2e^2\right)\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{3\sqrt{2}\sqrt{2+3x^2+x^4}}$$

Result (type 4, 127 leaves):

$$\frac{1}{3\sqrt{2+3x^2+x^4}}\left(e^2 x\left(2+3x^2+x^4\right)-6\,i\,(d-e)\,e\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]-i\left(3d^2-6de+4e^2\right)\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 396: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{d+e x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 122 leaves, 3 steps):

$$\frac{e\,x\,(2+x^2)}{\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\,e\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2+3x^2+x^4}}+\frac{d\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,\sqrt{2+3x^2+x^4}}$$

Result (type 4, 73 leaves) :

$$-\frac{1}{\sqrt{2+3x^2+x^4}}\,\mathrm{i}\sqrt{1+x^2}\,\sqrt{2+x^2}\left(e\,\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+(d-e)\,\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(d+e\,x^2)\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 124 leaves, 4 steps) :

$$\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,(d-e)\sqrt{2+3x^2+x^4}}-\frac{e\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticPi}\left[1-\frac{e}{d},\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,d\,(d-e)\sqrt{2+3x^2+x^4}}$$

Result (type 4, 59 leaves) :

$$-\frac{\mathrm{i}\sqrt{1+x^2}\,\sqrt{2+x^2}\,\text{EllipticPi}\left[\frac{2e}{d},\mathrm{i}\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]}{d\sqrt{2+3x^2+x^4}}$$

Problem 398: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{(d+e\,x^2)^2\sqrt{2+3x^2+x^4}}\,dx$$

Optimal (type 4, 316 leaves, 9 steps) :

$$-\frac{e\,x\,\left(2+x^2\right)}{2\,d\,\left(d^2-3\,d\,e+2\,e^2\right)\,\sqrt{2+3\,x^2+x^4}}+$$

$$\frac{e^2\,x\,\sqrt{2+3\,x^2+x^4}}{2\,d\,\left(d^2-3\,d\,e+2\,e^2\right)\,\left(d+e\,x^2\right)}+\frac{e\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{1+x^2}}\;\text{EllipticE}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{\sqrt{2}\,d\,(d-2\,e)\,(d-e)\,\sqrt{2+3\,x^2+x^4}}+$$

$$\frac{(2\,d-e)\,\left(1+x^2\right)\,\sqrt{\frac{2+x^2}{2+2\,x^2}}\;\text{EllipticF}\left[\text{ArcTan}[x],\frac{1}{2}\right]}{2\,d\,(d-e)^2\,\sqrt{2+3\,x^2+x^4}}-$$

$$\frac{e\,\left(3\,d^2-6\,d\,e+2\,e^2\right)\,\left(2+x^2\right)\,\text{EllipticPi}\left[1-\frac{e}{d},\text{ArcTan}[x],\frac{1}{2}\right]}{2\,\sqrt{2}\,d^2\,(d-2\,e)\,(d-e)^2\,\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3\,x^2+x^4}}$$

Result (type 4, 175 leaves) :

$$\frac{1}{2\,d\,\sqrt{2+3\,x^2+x^4}}\left(\frac{e^2\,x\,\left(2+3\,x^2+x^4\right)}{\left(d^2-3\,d\,e+2\,e^2\right)\,\left(d+e\,x^2\right)}+\left(i\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,\left(d\,e\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+d\,(d-e)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]+\left(-3\,d^2+6\,d\,e-2\,e^2\right)\,\text{EllipticPi}\left[\frac{2\,e}{d},i\,\text{ArcSinh}\left[\frac{x}{\sqrt{2}}\right],2\right]\right)\right)\Big/\left(d\,(d-2\,e)\,(d-e)\right)\right)$$

Problem 400: Result more than twice size of optimal antiderivative.

$$\int\left(c+e\,x^2\right)^3\,\left(a+c\,x^2+b\,x^4\right)^p\,dx$$

Optimal (type 6, 498 leaves, 8 steps) :

$$\frac{c\, e^2 \left(21 b - 5 e + 12 b p - 2 e p\right) x \left(a + c x^2 + b x^4\right)^{1+p}}{b^2 (5+4p)(7+4p)} + \frac{e^3 x^3 \left(a + c x^2 + b x^4\right)^{1+p}}{b (7+4p)} +$$

$$\Bigg(c \left(a e^3 (5+2p) - 3 a b e^2 (7+4p) + b^2 c^2 \left(35 + 48 p + 16 p^2\right)\right) x \left(1 + \frac{2 b x^2}{c - \sqrt{-4 a b + c^2}}\right)^{-p} \left(1 + \frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}\right)^{-p} \left(a + c x^2 + b x^4\right)^p \text{AppellF1}\left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 b x^2}{c - \sqrt{-4 a b + c^2}}, -\frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}\right]\Bigg) \Big/ \left(b^2 (5+4p)(7+4p)\right) +$$

$$\Bigg(e \left(c^2 e^2 \left(15 + 16 p + 4 p^2\right) + 3 b^2 c^2 \left(35 + 48 p + 16 p^2\right) - 3 b e \left(a e (5+4p) + c^2 \left(21 + 26 p + 8 p^2\right)\right)\right) x^3 \left(1 + \frac{2 b x^2}{c - \sqrt{-4 a b + c^2}}\right)^{-p} \left(1 + \frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}\right)^{-p} \left(a + c x^2 + b x^4\right)^p \text{AppellF1}\left[\frac{3}{2}, -p, -p, \frac{5}{2}, -\frac{2 b x^2}{c - \sqrt{-4 a b + c^2}}, -\frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}\right]\Bigg) \Big/ \left(3 b^2 (5+4p)(7+4p)\right)$$

Result (type 6, 1871 leaves) :

$$\Bigg(3 \times 4^{-1-p} c^3 \left(c + \sqrt{-4 a b + c^2}\right) x \left(\frac{c - \sqrt{-4 a b + c^2}}{2 b} + x^2\right)^{-p} \left(\frac{c + \sqrt{-4 a b + c^2}}{2 b} + x^2\right)^{-p} \left(\frac{c - \sqrt{-4 a b + c^2} + 2 b x^2}{b}\right)^{1+p} \left(\frac{c + \sqrt{-4 a b + c^2} + 2 b x^2}{b}\right)^{-1+p} \left(-2 a + \left(-c + \sqrt{-4 a b + c^2}\right) x^2\right)^2 \left(a + c x^2 + b x^4\right)^{-1+p} \text{AppellF1}\left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}, \frac{2 b x^2}{-c + \sqrt{-4 a b + c^2}}\right]\Bigg) \Big/$$

$$\Bigg(\left(-c + \sqrt{-4 a b + c^2}\right) \Bigg(-3 a \,\text{AppellF1}\left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}, \frac{2 b x^2}{-c + \sqrt{-4 a b + c^2}}\right] + p x^2 \Bigg(\left(-c + \sqrt{-4 a b + c^2}\right) \text{AppellF1}\left[\frac{3}{2}, 1 - p, -p, \frac{5}{2}, -\frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}, \frac{2 b x^2}{-c + \sqrt{-4 a b + c^2}}\right] - \left(c + \sqrt{-4 a b + c^2}\right) \text{AppellF1}\left[\frac{3}{2}, -p, 1 - p, \frac{5}{2}, -\frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}, \frac{2 b x^2}{-c + \sqrt{-4 a b + c^2}}\right]\Bigg)\Bigg)\Bigg) +$$

$$\Bigg(5 \times 2^{-2-p} b c^2 \left(c + \sqrt{-4 a b + c^2}\right) e x^3 \left(\frac{c - \sqrt{-4 a b + c^2}}{2 b} + x^2\right)^{-p} \left(\frac{c - \sqrt{-4 a b + c^2} + 2 b x^2}{b}\right)^{1+p} \left(-2 a + \left(-c + \sqrt{-4 a b + c^2}\right) x^2\right)^2 \left(a + c x^2 + b x^4\right)^{-1+p} \text{AppellF1}\left[\frac{3}{2}, -p, -p, \frac{5}{2}, -\frac{2 b x^2}{c + \sqrt{-4 a b + c^2}}, \frac{2 b x^2}{-c + \sqrt{-4 a b + c^2}}\right]\Bigg) \Big/$$

$$\Bigg(\left(-c + \sqrt{-4 a b + c^2}\right) \left(c + \sqrt{-4 a b + c^2} + 2 b x^2\right)$$

$$\left(-5\,a\,\text{AppellF1}\left[\frac{3}{2},\,-p,\,-p,\,\frac{5}{2},\,-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+\right.$$

$$p\,x^2\left(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{5}{2},\,1-p,\,-p,\,\frac{7}{2},\right.\right.$$

$$\left.-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]-\left(c+\sqrt{-4\,a\,b+c^2}\right)$$

$$\left.\left.\left.\text{AppellF1}\left[\frac{5}{2},\,-p,\,1-p,\,\frac{7}{2},\,-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\right)\right)+$$

$$\left(21\times 2^{-2-p}\,b\,c\left(c+\sqrt{-4\,a\,b+c^2}\right)e^2\,x^5\left(\frac{c-\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{1+p}\right.$$

$$\left(-2\,a+\left(-c+\sqrt{-4\,a\,b+c^2}\right)x^2\right)^2\left(a+c\,x^2+b\,x^4\right)^{-1+p}$$

$$\left.\text{AppellF1}\left[\frac{5}{2},\,-p,\,-p,\,\frac{7}{2},\,-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\Big/$$

$$\left(5\left(-c+\sqrt{-4\,a\,b+c^2}\right)\left(c+\sqrt{-4\,a\,b+c^2}+2\,b\,x^2\right)\right.$$

$$\left(-7\,a\,\text{AppellF1}\left[\frac{5}{2},\,-p,\,-p,\,\frac{7}{2},\,-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+\right.$$

$$p\,x^2\left(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{7}{2},\,1-p,\,-p,\,\frac{9}{2},\right.\right.$$

$$\left.-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]-\left(c+\sqrt{-4\,a\,b+c^2}\right)$$

$$\left.\left.\left.\text{AppellF1}\left[\frac{7}{2},\,-p,\,1-p,\,\frac{9}{2},\,-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\right)\right)+$$

$$\left(9\times 2^{-2-p}\,b\left(c+\sqrt{-4\,a\,b+c^2}\right)e^3\,x^7\left(\frac{c-\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{1+p}\right.$$

$$\left(-2\,a+\left(-c+\sqrt{-4\,a\,b+c^2}\right)x^2\right)^2\left(a+c\,x^2+b\,x^4\right)^{-1+p}$$

$$\left.\text{AppellF1}\left[\frac{7}{2},\,-p,\,-p,\,\frac{9}{2},\,-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\Big/$$

$$\left(7\left(-c+\sqrt{-4\,a\,b+c^2}\right)\left(c+\sqrt{-4\,a\,b+c^2}+2\,b\,x^2\right)\right.$$

$$\left(-9\,a\,\text{AppellF1}\left[\frac{7}{2},\,-p,\,-p,\,\frac{9}{2},\,-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+\right.$$

$$p\,x^2\left(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{9}{2},\,1-p,\,-p,\,\frac{11}{2},\right.\right.$$

$$\left.-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\,\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]-\left(c+\sqrt{-4\,a\,b+c^2}\right)$$

$$\text{AppellF1}\left[\frac{9}{2}, -p, 1-p, \frac{11}{2}, -\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}, \frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\Bigg)\Bigg)\Bigg)$$

Problem 401: Result more than twice size of optimal antiderivative.

$$\int \left(c+e\,x^2\right)^2 \left(a+c\,x^2+b\,x^4\right)^p \,dx$$

Optimal (type 6, 358 leaves, 7 steps):

$$\frac{e^2\,x\,\left(a+c\,x^2+b\,x^4\right)^{1+p}}{b\,(5+4\,p)} - \frac{1}{b\,(5+4\,p)} \left(a\,e^2-b\,c^2\,(5+4\,p)\right)\,x\,\left(1+\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}\right)^{-p} \left(1+\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right)^{-p} \left(a+c\,x^2+b\,x^4\right)^p$$
$$\text{AppellF1}\left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}, -\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right] + \frac{1}{3\,b\,(5+4\,p)}$$
$$c\,e\,\left(10\,b-3\,e+8\,b\,p-2\,e\,p\right)\,x^3\,\left(1+\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}\right)^{-p} \left(1+\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right)^{-p}$$
$$\left(a+c\,x^2+b\,x^4\right)^p\,\text{AppellF1}\left[\frac{3}{2}, -p, -p, \frac{5}{2}, -\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}, -\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right]$$

Result (type 6, 1001 leaves):

$$\frac{1}{15}\times 2^{-3-p}\left(c+\sqrt{-4ab+c^2}\right)x\left(\frac{c-\sqrt{-4ab+c^2}}{2b}+x^2\right)^{-p}$$

$$\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{b}\right)^{1+p}\left(-2a+\left(-c+\sqrt{-4ab+c^2}\right)x^2\right)\left(a+cx^2+bx^4\right)^{-1+p}$$

$$\left(-\left(\left(45c^2\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\right/\right.$$

$$\left(3a\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+\right.$$

$$px^2\left(\left(c-\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{3}{2},1-p,-p,\frac{5}{2},\right.\right.$$

$$\left.-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+\left(c+\sqrt{-4ab+c^2}\right)$$

$$\left.\left.\left.\text{AppellF1}\left[\frac{3}{2},-p,1-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\right)\right)+$$

$$ex^2\left(-\left(\left(50c\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\right/\right.$$

$$\left(5a\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+\right.$$

$$px^2\left(\left(c-\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{5}{2},1-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\right.\right.$$

$$\left.\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+\left(c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{5}{2},-p,\right.$$

$$\left.\left.\left.\left.1-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\right)\right)+$$

$$\left(21ex^2\,\text{AppellF1}\left[\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\Big/$$

$$\left(-7a\,\text{AppellF1}\left[\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]+\right.$$

$$px^2\left(\left(-c+\sqrt{-4ab+c^2}\right)\text{AppellF1}\left[\frac{7}{2},1-p,-p,\frac{9}{2},\right.\right.$$

$$\left.-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]-\left(c+\sqrt{-4ab+c^2}\right)$$

$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{7}{2},-p,1-p,\frac{9}{2},-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right]\right)\right)\right)\right)$$

Problem 402: Result more than twice size of optimal antiderivative.

$$\int\left(c+ex^2\right)\left(a+cx^2+bx^4\right)^p\,dx$$

Optimal (type 6, 274 leaves, 6 steps):

$$c\,x\left(1+\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}\right)^{-p}\left(1+\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right)^{-p}\left(a+c\,x^2+b\,x^4\right)^p\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right]+\frac{1}{3}\,e\,x^3\left(1+\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}\right)^{-p}\left(1+\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right)^{-p}\left(a+c\,x^2+b\,x^4\right)^p\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right]$$

Result (type 6, 706 leaves):

$$\frac{1}{3}\times 2^{-3-p}\left(c+\sqrt{-4\,a\,b+c^2}\right)x\left(\frac{c-\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c-\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{1+p}\left(-2\,a+\left(-c+\sqrt{-4\,a\,b+c^2}\right)x^2\right)\left(a+c\,x^2+b\,x^4\right)^{-1+p}\left(-\left(\left(9\,c\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\Big/\left(3\,a\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+p\,x^2\left(\left(c-\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{3}{2},1-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+\left(c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{3}{2},-p,1-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\right)\right)+\left(5\,e\,x^2\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\Big/\left(-5\,a\,\text{AppellF1}\left[\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+p\,x^2\left(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{5}{2},1-p,-p,\frac{7}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]-\left(c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{5}{2},-p,1-p,\frac{7}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\right)\right)$$

Problem 403: Result more than twice size of optimal antiderivative.

$$\int\left(a+c\,x^2+b\,x^4\right)^p\,dx$$

Optimal (type 6, 133 leaves, 2 steps):

$$x\left(1+\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}}\right)^{-p}\left(1+\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right)^{-p}\left(a+c\,x^2+b\,x^4\right)^p$$
$$\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c-\sqrt{-4\,a\,b+c^2}},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}}\right]$$

Result (type 6, 487 leaves):

$$\left(3\times 4^{-1-p}\left(c+\sqrt{-4\,a\,b+c^2}\right)x\left(\frac{c-\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\left(\frac{c+\sqrt{-4\,a\,b+c^2}}{2\,b}+x^2\right)^{-p}\right.$$
$$\left(\frac{c-\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{1+p}\left(\frac{c+\sqrt{-4\,a\,b+c^2}+2\,b\,x^2}{b}\right)^{-1+p}\left(-2\,a+\left(-c+\sqrt{-4\,a\,b+c^2}\right)x^2\right)^2$$
$$\left.\left(a+c\,x^2+b\,x^4\right)^{-1+p}\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\Big/$$
$$\left(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\left(-3\,a\,\text{AppellF1}\left[\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]+\right.\right.$$
$$p\,x^2\left(\left(-c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{3}{2},1-p,-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]-\right.$$
$$\left.\left.\left.\left(c+\sqrt{-4\,a\,b+c^2}\right)\text{AppellF1}\left[\frac{3}{2},-p,1-p,\frac{5}{2},-\frac{2\,b\,x^2}{c+\sqrt{-4\,a\,b+c^2}},\frac{2\,b\,x^2}{-c+\sqrt{-4\,a\,b+c^2}}\right]\right)\right)\right)$$

Problem 406: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{f+g\,x}{(d+e\,x)\,\sqrt{a+c\,x^4}}\,dx$$

Optimal (type 4, 446 leaves, 8 steps):

$$\frac{(e\,f-d\,g)\,\text{ArcTan}\left[\frac{\sqrt{-c\,d^4-a\,e^4}\,x}{d\,e\,\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{-c\,d^4-a\,e^4}}-\frac{(e\,f-d\,g)\,\text{ArcTanh}\left[\frac{a\,e^2+c\,d^2\,x^2}{\sqrt{c\,d^4+a\,e^4}\,\sqrt{a+c\,x^4}}\right]}{2\,\sqrt{c\,d^4+a\,e^4}}+$$
$$\left(\left(\sqrt{c}\,d\,f+\sqrt{a}\,e\,g\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(2\,a^{1/4}\,c^{1/4}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+c\,x^4}\right)-$$
$$\left(\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)(e\,f-d\,g)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$
$$\left.\left.\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\,\sqrt{a}\,\sqrt{c}\,d^2\,e^2},2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,a^{1/4}\,c^{1/4}\,d\,e\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+c\,x^4}\right)$$

Result (type 4, 275 leaves):

$$\frac{1}{2\,e\,\sqrt{a+c\,x^4}}\left(-\frac{2\,i\,g\,\sqrt{1+\frac{c\,x^4}{a}}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}\;x\right],-1\right]}{\sqrt{\frac{i\,\sqrt{c}}{\sqrt{a}}}}+\left(\left(-e\,f+d\,g\right)\left(2\,\left(-1\right)^{1/4}\,a^{1/4}\,\sqrt{c\,d^4+a\,e^4}\;\sqrt{1+\frac{c\,x^4}{a}}\;\text{EllipticPi}\left[\frac{i\,\sqrt{a}\;e^2}{\sqrt{c}\;d^2},\text{ArcSin}\left[\frac{\left(-1\right)^{3/4}\,c^{1/4}\,x}{a^{1/4}}\right],-1\right]+c^{1/4}\,d\,e\,\sqrt{a+c\,x^4}\;\left(-\text{Log}\left[-d^2+e^2\,x^2\right]+\text{Log}\left[a\,e^2+c\,d^2\,x^2+\sqrt{c\,d^4+a\,e^4}\;\sqrt{a+c\,x^4}\;\right]\right)\right)\right)\Big/\left(c^{1/4}\,d\,\sqrt{c\,d^4+a\,e^4}\right)\right)$$

Problem 407: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{f+g\,x}{\left(d+e\,x\right)\,\sqrt{-a+c\,x^4}}\,dx$$

Optimal (type 4, 218 leaves, 10 steps):

$$\frac{\left(e\,f-d\,g\right)\,\text{ArcTanh}\left[\frac{a\,e^2-c\,d^2\,x^2}{\sqrt{c\,d^4-a\,e^4}\;\sqrt{-a+c\,x^4}}\right]}{2\,\sqrt{c\,d^4-a\,e^4}}+\frac{a^{1/4}\,g\,\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{c^{1/4}\,e\,\sqrt{-a+c\,x^4}}+$$
$$\frac{a^{1/4}\,\left(e\,f-d\,g\right)\,\sqrt{1-\frac{c\,x^4}{a}}\;\text{EllipticPi}\left[\frac{\sqrt{a}\;e^2}{\sqrt{c}\;d^2},\text{ArcSin}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],-1\right]}{c^{1/4}\,d\,e\,\sqrt{-a+c\,x^4}}$$

Result (type 4, 719 leaves):

$$\frac{1}{\sqrt{-a+c x^4}}\left(-\frac{i g \sqrt{1-\frac{c x^4}{a}}\ \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\ x\right],-1\right]}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\ e}+\left(i f \left(a^{1/4}-i c^{1/4} x\right)^2 \sqrt{-\frac{(1-i) \left(a^{1/4}-c^{1/4} x\right)}{i a^{1/4}+c^{1/4} x}} \sqrt{\frac{(1+i) \left(a^{1/4}+i c^{1/4} x\right) \left(a^{1/4}+c^{1/4} x\right)}{\left(a^{1/4}-i c^{1/4} x\right)^2}} \left(\left(-c^{1/4} d+a^{1/4} e\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(1+i) \left(a^{1/4}+c^{1/4} x\right)}{2 i a^{1/4}+2 c^{1/4} x}}\right],2\right]-(1-i) a^{1/4} e\ \text{EllipticPi}\left[\frac{(1-i) \left(c^{1/4} d-i a^{1/4} e\right)}{c^{1/4} d-a^{1/4} e},\text{ArcSin}\left[\sqrt{\frac{(1+i) \left(a^{1/4}+c^{1/4} x\right)}{2 i a^{1/4}+2 c^{1/4} x}}\right],2\right]\right)\right)\Big/\left(a^{1/4} \left(-c^{1/4} d+a^{1/4} e\right) \left(i c^{1/4} d+a^{1/4} e\right)\right)+\left(d g \left(a^{1/4}-i c^{1/4} x\right)^2 \sqrt{-\frac{(1-i) \left(a^{1/4}-c^{1/4} x\right)}{i a^{1/4}+c^{1/4} x}} \sqrt{\frac{(1+i) \left(a^{1/4}+i c^{1/4} x\right) \left(a^{1/4}+c^{1/4} x\right)}{\left(a^{1/4}-i c^{1/4} x\right)^2}} \left(i \left(c^{1/4} d-a^{1/4} e\right) \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{(1+i) \left(a^{1/4}+c^{1/4} x\right)}{2 i a^{1/4}+2 c^{1/4} x}}\right],2\right]+(1+i) a^{1/4} e\ \text{EllipticPi}\left[\frac{(1-i) \left(c^{1/4} d-i a^{1/4} e\right)}{c^{1/4} d-a^{1/4} e},\text{ArcSin}\left[\sqrt{\frac{(1+i) \left(a^{1/4}+c^{1/4} x\right)}{2 i a^{1/4}+2 c^{1/4} x}}\right],2\right]\right)\right)\Big/\left(a^{1/4} e \left(-c^{1/4} d+a^{1/4} e\right) \left(i c^{1/4} d+a^{1/4} e\right)\right)\right)$$

Problem 408: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right) \sqrt{-4+4 \sqrt{3} x^2+x^4}} \, dx$$

Optimal (type 3, 65 leaves, 2 steps):

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\ \text{ArcTanh}\left[\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\ \sqrt{-4+4\sqrt{3}\ x^2+x^4}}\right]$$

Result (type 4, 685 leaves) :

$$\left(\left(-1+\sqrt{3}+x\right)^2 \sqrt{2\left(1+\sqrt{3}\right)-2\left(2+\sqrt{3}\right)x+\left(-1+\sqrt{3}\right)x^2-x^3} \sqrt{\frac{1+\sqrt{3}-\frac{4}{-1+\sqrt{3}+x}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}}\right.$$

$$\left(\left(i\left(-1+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}\right)+\frac{2\left(2\,i\sqrt{3}-\sqrt{2\left(2+\sqrt{3}\right)}+\sqrt{6\left(2+\sqrt{3}\right)}\right)}{-1+\sqrt{3}+x}\right)\right.$$

$$\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}+i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\frac{2\,i\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}\right]+$$

$$2\sqrt{6}\sqrt{\frac{4+2\sqrt{3}+x^2}{\left(-1+\sqrt{3}+x\right)^2}}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}$$

$$\text{EllipticPi}\left[\frac{2\sqrt{2\left(2+\sqrt{3}\right)}}{\sqrt{2\left(2+\sqrt{3}\right)}+i\left(3+\sqrt{3}\right)},\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}}{2^{3/4}\left(2+\sqrt{3}\right)^{1/4}}\right],\frac{2\,i\sqrt{2\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+i\sqrt{2\left(2+\sqrt{3}\right)}}\right]\right)\right)\Big/$$

$$\left(\left(\sqrt{2\left(2+\sqrt{3}\right)}+i\left(3+\sqrt{3}\right)\right)\sqrt{1+\sqrt{3}-\left(2+\sqrt{3}\right)x+\frac{1}{2}\left(-1+\sqrt{3}\right)x^2-\frac{x^3}{2}}\right.$$

$$\left.\sqrt{-4+4\sqrt{3}\,x^2+x^4}\sqrt{\sqrt{2\left(2+\sqrt{3}\right)}-i\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+x}\right)}\right)$$

Problem 409: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx$$

Optimal (type 3, 63 leaves, 2 steps):

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\ \text{ArcTan}\left[\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\ \sqrt{-4-4\sqrt{3}\,x^2+x^4}}\right]$$

Result (type 4, 1137 leaves):

$$-\Bigg(\Bigg(\left(-1-\sqrt{3}+x\right)^2\sqrt{\frac{-1+\sqrt{3}+\frac{4}{-1-\sqrt{3}+x}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}$$

$$\sqrt{\left(-24+16\sqrt{3}+\left(20-8\sqrt{3}\right)\left(1-\sqrt{3}+x\right)+\left(-2+4\sqrt{3}\right)\left(1-\sqrt{3}+x\right)^2+\left(1-\sqrt{3}+x\right)^3\right)}$$

$$\Bigg(\Bigg(i\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+x}}+$$

$$i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+x}}+$$

$$\sqrt{\left(-2i+2i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}\right)}+$$

$$\frac{1}{-1-\sqrt{3}+x}2\Bigg(2i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+x}}+$$

$$\sqrt{6}\sqrt{\left(-i+i\sqrt{3}-\sqrt{12-6\sqrt{3}}+2\sqrt{4-2\sqrt{3}}-\frac{8i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}\right)}+$$

$$\sqrt{\left(-2i+2i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+x}\right)}\Bigg)\Bigg)\Bigg)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]+$$

$$2\sqrt{6}\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+x}}\sqrt{1+\frac{8}{\left(-1-\sqrt{3}+x\right)^2}+\frac{2\left(1+\sqrt{3}\right)}{-1-\sqrt{3}+x}}$$

$$\text{EllipticPi}\left[\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)},\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\right],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+i\left(-3+\sqrt{3}\right)}\right]\right)\right)\Bigg/$$

$$\left(\left(\sqrt{4-2\sqrt{3}}-i\left(-3+\sqrt{3}\right)\right)\sqrt{\sqrt{4-2\sqrt{3}}-i\left(1+\sqrt{3}\right)-\frac{8 i}{-1-\sqrt{3}+x}}\right.$$

$$\sqrt{\left(8\left(1+\sqrt{3}\right)+4\left(3+\sqrt{3}\right)\left(-1-\sqrt{3}+x\right)+2\left(1+\sqrt{3}\right)\left(-1-\sqrt{3}+x\right)^2+\frac{1}{2}\left(-1-\sqrt{3}+x\right)^3\right)}$$

$$\sqrt{\left(48-32\sqrt{3}-64\left(1-\sqrt{3}+x\right)+32\sqrt{3}\left(1-\sqrt{3}+x\right)+24\left(1-\sqrt{3}+x\right)^2-\right.}$$

$$\left.\left.16\sqrt{3}\left(1-\sqrt{3}+x\right)^2-4\left(1-\sqrt{3}+x\right)^3+4\sqrt{3}\left(1-\sqrt{3}+x\right)^3+\left(1-\sqrt{3}+x\right)^4\right)\right)$$

Problem 410: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}\,x^2+4x^4}}\,dx$$

Optimal (type 3, 72 leaves, 2 steps):

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\;\text{ArcTanh}\left[\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-1+4\sqrt{3}\,x^2+4x^4}}\right]$$

Result (type 4, 623 leaves):

$$\left(\left(-1+\sqrt{3}+2\,x\right)^2\sqrt{\frac{1+\sqrt{3}-\frac{4}{-1+\sqrt{3}+2\,x}}{3+\sqrt{3}+\mathbb{i}\,\sqrt{2\,\left(2+\sqrt{3}\right)}}}\right.$$

$$\left(\left(\mathbb{i}\,\left(-1+\sqrt{3}+\mathbb{i}\,\sqrt{2\,\left(2+\sqrt{3}\right)}\right)+\frac{2\,\left(2\,\mathbb{i}\,\sqrt{3}-\sqrt{2\,\left(2+\sqrt{3}\right)}+\sqrt{6\,\left(2+\sqrt{3}\right)}\right)}{-1+\sqrt{3}+2\,x}\right)\right.$$

$$\sqrt{\sqrt{2\,\left(2+\sqrt{3}\right)}+\mathbb{i}\,\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\,\left(2+\sqrt{3}\right)}-\mathbb{i}\,\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}}{2^{3/4}\,\left(2+\sqrt{3}\right)^{1/4}}\right],\,\frac{2\,\mathbb{i}\,\sqrt{2\,\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+\mathbb{i}\,\sqrt{2\,\left(2+\sqrt{3}\right)}}\right]+$$

$$4\,\sqrt{3}\,\sqrt{\frac{2+\sqrt{3}+2\,x^2}{\left(-1+\sqrt{3}+2\,x\right)^2}}\,\sqrt{\sqrt{2\,\left(2+\sqrt{3}\right)}-\mathbb{i}\,\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}$$

$$\text{EllipticPi}\left[\frac{2\,\sqrt{2\,\left(2+\sqrt{3}\right)}}{\sqrt{2\,\left(2+\sqrt{3}\right)}+\mathbb{i}\,\left(3+\sqrt{3}\right)},\right.$$

$$\left.\left.\left.\text{ArcSin}\left[\frac{\sqrt{\sqrt{2\,\left(2+\sqrt{3}\right)}-\mathbb{i}\,\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}}{2^{3/4}\,\left(2+\sqrt{3}\right)^{1/4}}\right],\,\frac{2\,\mathbb{i}\,\sqrt{2\,\left(2+\sqrt{3}\right)}}{3+\sqrt{3}+\mathbb{i}\,\sqrt{2\,\left(2+\sqrt{3}\right)}}\right]\right)\right)\Big/$$

$$\left(\left(\sqrt{2\,\left(2+\sqrt{3}\right)}+\mathbb{i}\,\left(3+\sqrt{3}\right)\right)\sqrt{-2+8\,\sqrt{3}\,x^2+8\,x^4}\right.$$

$$\left.\sqrt{\sqrt{2\,\left(2+\sqrt{3}\right)}-\mathbb{i}\,\left(1-\sqrt{3}+\frac{8}{-1+\sqrt{3}+2\,x}\right)}\right)$$

Problem 411: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\,dx$$

Optimal (type 3, 70 leaves, 2 steps):

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\;\text{ArcTan}\left[\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\,\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\right]$$

Result (type 4, 1198 leaves):

$$-\Bigg(\Bigg(\Bigg(\left(-1-\sqrt{3}+2x\right)^2\sqrt{\frac{-1+\sqrt{3}+\frac{4}{-1-\sqrt{3}+2x}}{-3+\sqrt{3}-i\sqrt{4-2\sqrt{3}}}}$$

$$\sqrt{\left(-24+16\sqrt{3}+\left(20-8\sqrt{3}\right)\left(1-\sqrt{3}+2x\right)+\left(-2+4\sqrt{3}\right)\left(1-\sqrt{3}+2x\right)^2+\left(1-\sqrt{3}+2x\right)^3\right)}$$

$$\Bigg(\Bigg(i\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+2x}}+$$

$$i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+2x}}+$$

$$\sqrt{\left(-2i+2i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+2x}\right)}+$$

$$\frac{1}{-1-\sqrt{3}+2x}2\Bigg(2i\sqrt{3}\sqrt{\sqrt{4-2\sqrt{3}}+i\left(1+\sqrt{3}\right)+\frac{8i}{-1-\sqrt{3}+2x}}+$$

$$\sqrt{6}\sqrt{\left(-i+i\sqrt{3}-\sqrt{12-6\sqrt{3}}+2\sqrt{4-2\sqrt{3}}-\frac{8i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+2x}\right)}+$$

$$\sqrt{\left(-2i+2i\sqrt{3}-2\sqrt{12-6\sqrt{3}}+4\sqrt{4-2\sqrt{3}}-\frac{16i\left(-2+\sqrt{3}\right)}{-1-\sqrt{3}+2x}\right)}\Bigg)\Bigg)\;\text{EllipticF}\Big[$$

$$\text{ArcSin}\Big[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-\mathrm{i}\left(1+\sqrt{3}\right)-\frac{8\,\mathrm{i}}{-1-\sqrt{3}+2x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\Big],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+\mathrm{i}\left(-3+\sqrt{3}\right)}\Big]+$$

$$2\sqrt{6}\sqrt{\sqrt{4-2\sqrt{3}}-\mathrm{i}\left(1+\sqrt{3}\right)-\frac{8\,\mathrm{i}}{-1-\sqrt{3}+2x}}\sqrt{1+\frac{8}{\left(-1-\sqrt{3}+2x\right)^2}+\frac{2\left(1+\sqrt{3}\right)}{-1-\sqrt{3}+2x}}$$

$$\text{EllipticPi}\Big[\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}-\mathrm{i}\left(-3+\sqrt{3}\right)},$$

$$\text{ArcSin}\Big[\frac{\sqrt{\sqrt{4-2\sqrt{3}}-\mathrm{i}\left(1+\sqrt{3}\right)-\frac{8\,\mathrm{i}}{-1-\sqrt{3}+2x}}}{2^{3/4}\left(2-\sqrt{3}\right)^{1/4}}\Big],\frac{2\sqrt{4-2\sqrt{3}}}{\sqrt{4-2\sqrt{3}}+\mathrm{i}\left(-3+\sqrt{3}\right)}\Big]\Bigg)\Bigg)\Big/$$

$$\Bigg(2\left(\sqrt{4-2\sqrt{3}}-\mathrm{i}\left(-3+\sqrt{3}\right)\right)\sqrt{\sqrt{4-2\sqrt{3}}-\mathrm{i}\left(1+\sqrt{3}\right)-\frac{8\,\mathrm{i}}{-1-\sqrt{3}+2x}}$$

$$\sqrt{\Big(8\left(1+\sqrt{3}\right)+4\left(3+\sqrt{3}\right)\left(-1-\sqrt{3}+2x\right)+}$$

$$\overline{2\left(1+\sqrt{3}\right)\left(-1-\sqrt{3}+2x\right)^2+\frac{1}{2}\left(-1-\sqrt{3}+2x\right)^3\Big)}$$

$$\sqrt{\Big(12-8\sqrt{3}-16\left(1-\sqrt{3}+2x\right)+8\sqrt{3}\left(1-\sqrt{3}+2x\right)+6\left(1-\sqrt{3}+2x\right)^2-}$$

$$\overline{4\sqrt{3}\left(1-\sqrt{3}+2x\right)^2-\left(1-\sqrt{3}+2x\right)^3+\sqrt{3}\left(1-\sqrt{3}+2x\right)^3+\frac{1}{4}\left(1-\sqrt{3}+2x\right)^4\Big)}\Bigg)$$

Problem 412: Unable to integrate problem.

$$\int\frac{f+g\,x}{\left(d+e\,x\right)\sqrt{a+b\,x^2+c\,x^4}}\,dx$$

Optimal (type 4, 560 leaves, 8 steps):

$$\frac{(e f-d g)\,\text{ArcTan}\left[\frac{\sqrt{-c d^4-b d^2 e^2-a e^4}\,x}{d e\sqrt{a+b x^2+c x^4}}\right]}{2\sqrt{-c d^4-e^2\left(b d^2+a e^2\right)}}-\frac{(e f-d g)\,\text{ArcTanh}\left[\frac{b d^2+2 a e^2+\left(2 c d^2+b e^2\right) x^2}{2\sqrt{c d^4+b d^2 e^2+a e^4}\,\sqrt{a+b x^2+c x^4}}\right]}{2\sqrt{c d^4+b d^2 e^2+a e^4}}+\left(\left(\sqrt{c}\,d f+\sqrt{a}\,e g\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(2 a^{1/4} c^{1/4}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+b x^2+c x^4}\right)-\left(\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)(e f-d g)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\text{EllipticPi}\left[\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\sqrt{c}\,d^2 e^2},2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(4 a^{1/4} c^{1/4} d e\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{a+b x^2+c x^4}\right)$$

Result (type 8, 31 leaves):

$$\int\frac{f+g x}{(d+e x)\sqrt{a+b x^2+c x^4}}\,dx$$

Problem 413: Unable to integrate problem.

$$\int\frac{f+g x}{(d+e x)\sqrt{-a+b x^2+c x^4}}\,dx$$

Optimal (type 4, 527 leaves, 10 steps):

$$-\frac{(e f-d g)\,\text{ArcTanh}\left[\frac{b d^2-2 a e^2+\left(2 c d^2+b e^2\right) x^2}{2 \sqrt{c d^4+b d^2 e^2-a e^4}\,\sqrt{-a+b x^2+c x^4}}\right]}{2 \sqrt{c d^4+b d^2 e^2-a e^4}}+$$

$$\frac{\sqrt{b+\sqrt{b^2+4 a c}}\; g \left(1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}\right) \text{EllipticF}\left[\text{ArcTan}\left[\frac{\sqrt{2}\,\sqrt{c}\, x}{\sqrt{b+\sqrt{b^2+4 a c}}}\right],-\frac{2 \sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right]}{\sqrt{2}\,\sqrt{c}\; e \sqrt{\frac{1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}{1+\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}}\,\sqrt{-a+b x^2+c x^4}}+$$

$$\left(\sqrt{-b+\sqrt{b^2+4 a c}}\;(e f-d g)\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2+4 a c}}}\;\sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2+4 a c}}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(b-\sqrt{b^2+4 a c}\right) e^2}{2 c d^2},\text{ArcSin}\left[\frac{\sqrt{2}\,\sqrt{c}\, x}{\sqrt{-b+\sqrt{b^2+4 a c}}}\right],\frac{b-\sqrt{b^2+4 a c}}{b+\sqrt{b^2+4 a c}}\right]\right)\Big/$$

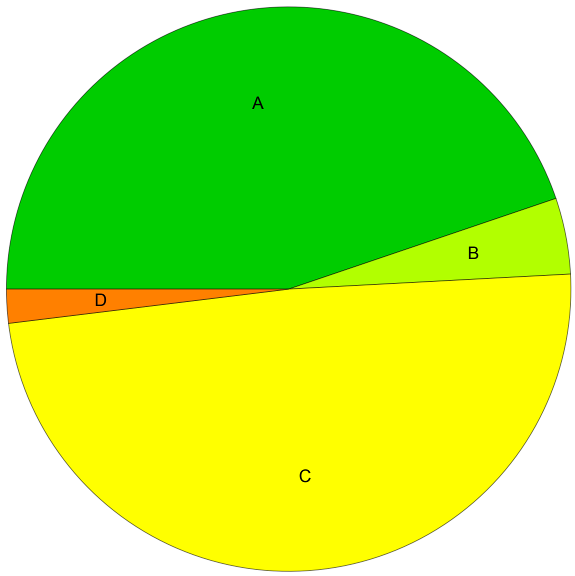
$$\left(\sqrt{2}\,\sqrt{c}\; d\, e \sqrt{-a+b x^2+c x^4}\right)$$

Result (type 8, 33 leaves):

$$\int \frac{f+g x}{(d+e x)\,\sqrt{-a+b x^2+c x^4}}\,dx$$

Summary of Integration Test Results

413 integration problems



A - 185 optimal antiderivatives

B - 18 more than twice size of optimal antiderivatives

C - 202 unnecessarily complex antiderivatives

D - 8 unable to integrate problems

E - 0 integration timeouts